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Physic  
Thermodynamics

# AN ELEMENTARY TREATISE

ON

# HEAT.

BY

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# H E A T .

## CHAPTER I.

ON TEMPERATURE AND ITS MEASUREMENT. THE MERCURIAL THERMOMETER. MAXIMUM AND MINIMUM THERMOMETERS.

1. **Temperature.** If we place a poker in a fire, and after some time remove it, it feels *hot*. The physical cause of this sensation is called *heat*. A piece of iron which has been exposed to the air in Britain will generally feel *cold*.

If we hold for some time in the right hand a piece of iron which feels hot, and then grasp the right hand with the left, the left hand will experience a sensation of heat, or the right hand will feel hot to the left, and at the same time the left hand will feel cold to the right. The same will be true if we hold for some time in the right hand any other body which feels hot. The fact that the right hand feels hot to the left after holding the hot iron, while it may have felt neither hot nor cold before, proves that the hand has received heat from the iron. We therefore infer that when a body feels hot to the hand, it imparts heat to the hand while touching it. It then follows that, because the right hand feels hot to the left hand, heat must pass from the right hand to the left. But the left hand feels cold to the right hand, and associating this sensation with what we have just proved ; viz., that heat passes from the right hand to the left, we infer that a body feels cold to the hand when heat passes from the hand to the body. We may arrive at the same conclusion by holding in the right hand a piece of very cold iron, after which the right hand will feel cold to the left, and the left hand warm to the right. Now, the fact that the left hand feels warm to the right, shows that heat passes

from it to the right hand ; and we infer that this also is the reason why the right hand feels cold to the left. Also, the fact that the right hand has been made to feel cold to the left by its contact with the iron, shows that heat must have left the hand and entered the iron, which accounts for the latter feeling cold to the hand.

2. We thus see that a body feels hot to the hand when heat passes from the body to the hand, and cold when it passes in the opposite direction. The sensations of heat and cold are therefore essentially *relative*. This may be further illustrated thus :—

Place the right hand in a mixture of ice and salt, and the left hand in hot water ; then place both in cold water ; the cold water will feel warm to the right hand and cold to the left, showing that the sensation of heat or cold depends on the condition of the hand relative to the hot or cold body, and not simply on the state of the body itself. The water in a bath sometimes feels warm to the hands and cold to the feet.

3. If a body feel hot to the hand, it is said to be at a higher temperature ; and if it feel cold, at a lower temperature, than the hand. Now, we have seen that a body feels hot or cold according as heat passes from it to the hand, or from the hand to it. The direction of the flow of heat then, in this case, determines whether the body or the hand is at the higher temperature ; and applying the same criterion in all cases to determine whether of two bodies is the hotter, we may define temperature thus :—

DEF. *The temperature of a body is its thermal condition with reference to its power of communicating heat to, or of receiving heat from, other bodies ; the body A being said to have a higher temperature than B if B gain heat from A when they are placed in contact.*

*Hence, if when two bodies are placed in contact neither of them gain heat at the expense of the other, the two bodies are said to be at the same temperature.*

In dealing with gravitation, one point is said to be at a higher level than another if a heavy body tend to move from the former point to the latter when a suitable path

has been prepared for it. If there is no tendency for a heavy body to move from either point to the other, the two points are said to be at the same level. Thus difference of temperature has the same relation to heat as difference of level has to gravitating bodies. There is a clear distinction between level and gravitating matter, even when the material is sustained at a high level by means of other material beneath it, as in the case of a stone on the top of a mountain, and there should be no more risk of confusing heat with temperature, even though heat be required to produce the temperature in a particular body, than of failing to distinguish between the mountain and its height.

4. When two bodies have the same temperature, they are said to be in *thermal equilibrium* with each other.

Now, *it is found, experimentally*, that if two bodies *A* and *C* are each in thermal equilibrium with a third body *B*, then if *A* and *C* be placed in contact, they will be in thermal equilibrium with each other, provided no chemical action take place between them. Hence we infer that *bodies which are in thermal equilibrium with the same body, are in thermal equilibrium with each other*; and our definition is consistent with itself when it directs us to assign to them all the same temperature, viz., that of the body with which they are all in thermal equilibrium. It is on this principle that the employment of thermometers depends. If two bodies act chemically upon one another, they may be put into thermal communication with each other by means of a diaphragm impervious to either body, but pervious to heat. If no heat be lost or gained by either from the other, the bodies are in thermal equilibrium; but it will be seen that unless radiation can take place between them, all we can say respecting them is that they are each in thermal equilibrium with the diaphragm, and this is no more than saying that they are in thermal equilibrium with a third body.

5. If we touch in succession several bodies, all of which feel hot, some may feel hotter than others; and similarly, if we touch several bodies, all of which feel cold, we may experience sensations of different degrees of coldness.

Now, if all the bodies are formed of the same material, and in the same mechanical condition, we are justified in assuming that, of two hot bodies, that is at the higher temperature which feels the hotter, and similarly for two cold bodies. But if the bodies be of different materials, we are no longer justified in making this assumption, and for this reason; viz., that the power of a hot body to produce the sensation of heat does not depend *entirely* on its *thermal* condition with reference to its power of communicating heat to other bodies (that is, on its temperature), but depends also on the rate at which it can transmit heat through its substance; and this is different for different materials. Thus a piece of iron and a hollow piece of wood may both feel hot, and the iron feel much hotter than the wood; yet if the iron be placed in the hollow of the wood for some time, it may then feel hotter than before, thus showing that heat has passed from the wood to the iron, and that the iron was therefore originally at a temperature *lower* than that of the wood. Again, if a piece of iron and a piece of wood be in contact for a long while in a hot place, the iron will feel much the hotter though it is in thermal equilibrium with the wood, and therefore at the same temperature as the latter. This is due to the difference in the conducting powers for heat of iron and wood, and will be discussed in a subsequent chapter. In fact, the intensity of the sensation of heat depends on the rate at which heat enters the skin, and not simply on the temperature of the body touched.

**6. The Thermometer.** In order, then, to determine whether of two bodies, *A* and *B*, is the hotter, we ought to place them in contact, and determine in which direction the heat passes; but as this is frequently a very difficult or impossible operation, recourse must be had to some more convenient method. This is generally accomplished by taking a third body *C*, very small compared with *A* or *B*, and called a *thermometer*. This is placed in contact with *A* till it is in thermal equilibrium with it, and therefore at the same temperature; and subsequently in contact with *B*, till it is again in thermal equilibrium, and therefore at the temperature of *B*; and it is ascer-



tained whether the temperature of  $C$  is higher when in equilibrium with  $A$  or with  $B$  by observation of some property of the body  $C$ , which changes with its temperature,—as, for example, its volume.

If the body  $C$  be in precisely the same thermal condition both when it is in thermal equilibrium with  $A$  and when in thermal equilibrium with  $B$ , it follows from Art. 4 that  $A$  and  $B$  are at the same temperature. Hence, a thermometer will enable us to determine whether a number of bodies are at the same temperature or not; and if not, to arrange them in the order of their respective temperatures.

Though the changes in volume of the thermometric substance are generally taken to indicate the variations of temperature which it experiences, we may avail ourselves of any other property of the thermometric substance which changes with temperature and which can be readily observed. Thus, in tempering steel, the colour exhibited by the thin film of oxide on its surface serves to indicate the temperature. Sometimes the electrical resistance of a platinum wire, which increases with the temperature, is employed to indicate the temperature to which the wire is exposed; or the thermo-electric current produced by a pair of dissimilar metals is employed to indicate the difference of temperature of the junctions of the metals. The *pitch* of the note of an organ pipe will indicate the temperature of the air; and as nearly all the physical properties of bodies change with the temperature, almost any property may be made available for thermometric purposes.

7. All bodies, with very few exceptions, among which may be mentioned iodide of silver, stretched caoutchouc (in the direction of the tension), Iceland spar and some other crystals in particular directions, water at or near the freezing point, and some alloys, expand when heated, and contract again on cooling; so that, other things being the same, their volumes increase as the temperature rises, and return to their original values when the bodies regain their original temperature; though glass presents a curious exception to the last statement, as its volume

does not return to its original value after expansion until a long time after the original temperature has been restored. Suppose it possible to measure accurately the volume of a given small quantity of some substance, as, for instance, mercury. Then, if the mercury be placed in contact with each of any number of bodies in succession, till it is in thermal equilibrium with it, and its volume be measured in each case, we can arrange the series of bodies in order of temperature.

8. **Scales of Temperature.** Suppose  $K$ ,  $L$ , and  $M$  to be three bodies, of which  $K$  is the coldest and  $M$  the hottest. Then, although we can arrange the bodies in order of temperature, by means of the small quantity of mercury, or other substance, whose volume we measure, yet we are not at present in a position to tell whether the difference between the temperatures of  $K$  and  $L$  is greater, equal to, or less than, the difference between the temperatures of  $L$  and  $M$ ; in fact we have as yet no graduated scale of temperature. Now, if our thermometer consist of a given mass of mercury, we may, if we please, define equal increments of temperature to be such as produce equal increments in the volume of the mercury, and it will then follow directly from the definition that the difference between the temperatures of two bodies is proportional to the difference in the volume of a given mass of mercury when in thermal equilibrium with the first and second body respectively. We are thus enabled to form an arbitrary *scale of temperature* which will depend upon the properties of the particular substance (in this case mercury) which forms our thermometer. As, however, the mercurial thermometer is used much more extensively than any other, before alluding to other scales of temperature we shall describe the construction and mode of graduation of the ordinary mercurial thermometer.

The problem of comparing differences of temperature in different parts of the scale, or of determining equal intervals of temperature, is comparable in difficulty with the corresponding problem in time. We meet with no such difficulty in the measurement of lengths, because the lengths which are to be compared may be superposed, or a third body of constant length may be superposed on

each in succession. It is obvious, however, that an interval of time of to-day cannot thus be compared with an interval to-morrow; for having once elapsed, it is gone for ever, and some other method of comparing intervals of time must be resorted to. The First Law of Motion indicates the criterion of equality in intervals of time. The principles of Thermodynamics teach us how to determine the equality of intervals of temperature, but until we have studied the action of heat engines we must be content with a scale of temperature derived from the properties of some particular substance or group of substances.

9. **The Mercurial Thermometer.** If we wish to detect and measure very small increments in the volume of a substance, it is obviously most convenient to employ, when possible, a fluid as the substance whose volume is to be changed, for we can then avail ourselves of a large volume of fluid contained in a suitable reservoir (*e.g.*, the bulb of the thermometer), while we can cause the free surface to be contained in a tube of very small bore, and thus an extremely small increase in the volume of the fluid will be accompanied by a considerable movement of the surface in the tube. In this case it is plain that the change of volume observed is only the difference between the change of volume of the fluid and of the envelope in which it is contained; but this is no serious inconvenience, since fluids can be found which expand very much more than glass, and it is of glass that the envelope is usually made: moreover, if we employed solid bars, we should have to take care that our measuring instruments did not themselves expand, or we should obtain no better result.

10. To construct a mercurial thermometer, a glass tube of very fine bore is taken, and a small bulb is blown at one end, while a cup is formed at the other extremity, as shown in Fig. 1. In this cup is placed a small quantity of mercury, and the bulb is then heated over a flame. The air in the bulb expands when heated, and a considerable portion of it escapes by bubbling up through the mercury in the cup. The bulb is then allowed to cool, when the pressure of the air within it diminishes (as

FIG. 1.



explained in Chapter IV.), and the pressure of the external air drives some of the mercury in the cup down the tube and into the bulb. If it be desired to take great care in the construction of the thermometer, this process is repeated until the bulb and a portion of the tube are filled with mercury when at the ordinary temperature; but in general a more expeditious process is adopted. A small portion of mercury having entered the bulb, as described above, the latter is heated until the mercury boils, when the heavy mercury vapour drives out the lighter air at the top of the tube, and, when the whole of the air has been expelled, the bulb and tube are filled with mercury vapour (except the space occupied by the liquid mercury remaining in the bulb). The instrument is then allowed to cool, and as the mercury vapour condenses, the mercury in the cup is forced into the tube by the pressure of the external air, and, except for a very small quantity of air which may have been left behind, the bulb and tube are completely filled with liquid mercury. The instrument is then heated to a temperature somewhat higher than the highest for which it is intended to be subsequently used, when the mercury expands so as to completely fill the tube, and drive out any residue of air which may have been left above it. The top of the tube is then sealed with the blowpipe, while the mercury remains hot. The instrument should be kept for some months after the bulb has been blown before it is graduated, because glass after being highly heated does not acquire its permanent volume for some time, and this is the reason why, in the filling of the best thermometers to be used for moderate temperatures only, the mercury is not boiled, but the air is driven from the bulb by heating it slightly many times in succession. (See Art. 7.)

The external diameter of the tube of a thermometer is generally very great compared with the diameter of the bore. Moreover, it is of importance for many purposes that the bulb should be constructed of very thin

glass. In most well-made thermometers the bulb is elongated so as to be almost cylindrical, while its external diameter is frequently not greater than that of the tube. The section of the bore of the tube is often flattened instead of circular, so that the mercury may expose a broad surface to the front of the thermometer though its sectional area remains very small; and frequently the bulb is not blown on the thermometer tube, but on a separate tube of the same kind of glass, and then fused to the tube of the instrument.

From what has just been said it will be seen that in the thermometer tube above the mercury there is scarcely any air, so that the interior of the bulb and tube is exposed only to the pressure of the mercury vapour and the liquid column. Therefore, unless the temperature is sufficiently high to cause a considerable pressure of mercury vapour, the pressure within the instrument is practically independent of the temperature to which it is exposed, so long as the thermometer is horizontal.

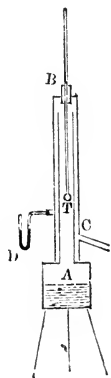
**11. Graduation of the Thermometer.** It remains to graduate the thermometer. It is found that under ordinary atmospheric pressure pure ice always melts at the same temperature, which is called the freezing point; and also that the temperature of the steam above the surface of boiling water depends only on the barometric pressure to which it is subjected. A pressure of 29.905\* inches of mercury at the freezing point in the latitude of London, and at the level of the sea, is generally taken as the standard pressure, and the temperature of the steam above the surface of water boiling under this

\* The pressure of the standard atmosphere is generally taken to be equal to that of 760 millimètres or 29.9215...inches of mercury at 0° C. at the sea-level in latitude 45°. On account of the variation of gravity with the latitude this is equivalent to the pressure exerted by 29.905 inches of mercury at the sea-level in the latitude of London, or by 29.898 inches of mercury in the latitude of Manchester. In order to avoid ambiguity arising from the variation of gravity, it has been proposed to adopt as the standard pressure a pressure of a megadyne per square centimètre. This is equivalent to 74.964 centimètres or 29.514 inches of mercury at 0° C. at the sea-level in the latitude of London. This would, however, necessitate a reconstruction of our thermometric scales, since the temperature of steam above water boiling under this pressure is only about 99.63° C.

pressure is always the same, and is called the boiling point. If the height of the barometer differ from this standard height, in graduating the thermometer a correction has to be made in consequence. (The correction amounts to about  $1.4^{\circ}$  F.\* for one inch of mercury.)

In order to graduate the instrument, it is first placed in melting ice, and a mark made on the tube at the height at which the surface of the mercury stands. This mark corresponds to the freezing point, and should always be determined before the boiling point, because, when it has been heated to the latter temperature, the glass does not at once contract to its proper volume, and, if the freezing point were determined immediately after the boiling point, the mark would after some time be found to be too far down the tube. In determining the freezing point, the thermometer should be immersed so far in the ice that the surface of the mercury is only just visible above it, in order that the *whole* of the mercury may have the temperature of the ice. In determining the freezing point, it is not generally necessary to observe the height of the barometer, for the melting point of ice is lowered by only  $\frac{1^{\circ}}{75}$  F. for an increase of pressure amounting to a whole atmosphere.

FIG. 2.



12. In order to determine the boiling point, the instrument shown in Fig. 2, and called a hypsometer, is employed. This instrument consists of a vessel *A*, containing water, the upper portion of which is a tube open at the top. Over this tube is placed the wider tube *BC*, which is closed at *B* by a perforated cork through which the stem of the thermometer *T* passes. *C* is a small tube for the exit of the steam, sloping downwards in order to allow the ready escape of condensed water. *D* is a small bent tube containing water, which is sometimes attached to the instrument to indicate if the pressure of the steam within exceed that of

\* For description of Fahrenheit's scale, see Art. 15.

the external air. When the water in *A* is made to boil, the thermometer soon becomes surrounded by an atmosphere of steam, which is itself surrounded by a steam-jacket, on account of the steam having to pass down between the tubes before escaping at *C*, and is thereby prevented from cooling. The thermometer is pushed through the cork at *B* till the surface of the mercury is only just visible above it, in order that the whole of the mercury may be exposed to the same temperature. The stem is then scratched at the level of the surface of the mercury (the height of the barometer being observed at the same time, in order to make any necessary correction for the pressure not being the standard pressure of 29.905 inches of mercury). The mark so placed upon the tube is called the *boiling point*.

The effect of the steam-jacket will be readily understood after reading Chapter VI. It will then be seen that while the effect of loss of heat from the outer surface of the jacket is to cause the steam in the jacket to lose heat, this will not cause any fall of temperature in the steam; for if it did, the steam would be unable to exist exposed to the atmospheric pressure. The effect will be simply to condense some portion of the steam without lowering the temperature; and thus the inner wall of the steam-jacket, that is the metal tube surrounding the thermometer, will retain the full temperature of the steam, and the thermometer will not lose heat by radiation to it. If no steam-jacket were employed, though the steam surrounding the thermometer would retain its full temperature, the metal tube would be chilled, and the thermometer would lose heat by radiation to it, and thus have a temperature below that of the steam around it.

13. It is important that the thermometer should be immersed in the steam and not in the water, for the temperature at which the water will boil depends on a great many conditions. For example, the material of which the vessel is composed affects the boiling point, and water will boil in a metallic vessel at a lower temperature than in a vessel made of glass. Again, saline impurities dissolved in the water will cause the boiling point to be

raised, but the temperature of the steam above the surface of the water depends only on the pressure.

If a quantity of common salt be dissolved in the water, the temperature of the solution may be raised about  $16^{\circ}$  F. above the boiling point, but the temperature of the steam will be the boiling point due to the existing pressure. If two thermometers be immersed in the steam above the solution of salt, one of which has its bulb naked and clean, while the other has a piece of cotton cord which has been dipped in a strong solution of calcium chloride twisted round the bulb, the naked thermometer will indicate  $212^{\circ}$  F., while the other may denote a temperature of  $227^{\circ}$  F., or even higher. At the same time, a third thermometer immersed in the solution will read  $228^{\circ}$  F. This experiment shows that care should be taken to see that the bulb of the thermometer is clean when immersed in the steam.

14. It is very important that when the fixed points are marked upon a thermometer tube, the tube should be in the same position relative to the vertical as that in which it is to be subsequently used. For if a thermometer be observed in a horizontal position, and be then turned into the vertical position, the reading will be lowered in consequence of the increased pressure in the bulb, and the effect will be greater the longer the column of mercury and the thinner the walls of the bulb.

As the air has no access to the interior of a thermometer it follows that the reading of a thermometer will depend to some extent on the barometric pressure; for the pressure of the air will compress the bulb of the thermometer, diminishing its volume by an amount depending on that pressure. This can be rendered very apparent by placing a thermometer in the receiver of an air-pump and exhausting the air, when the thermometer will continue depressed after sufficient time has been allowed for the interior of the receiver to regain its original temperature, the depression being due to the expansion of the glass under the diminished pressure of the air. When self-registering mercurial thermometers are employed for determining the temperature at considerable depths in



the sea, great care must be taken to protect the bulbs from the pressure to which they would otherwise be exposed.

15. It remains now to subdivide the stem of the thermometer between the freezing and boiling points into degrees, and to mark off degrees above the higher and below the lower of these temperatures. There are three different modes of doing this in general use in different parts of Europe; in other words, there are three distinct *scales of temperature*. In Fahrenheit's scale, which is chiefly adopted in England, the freezing point is marked  $32^{\circ}$ , and the boiling point  $212^{\circ}$ . The tube between these points is then divided into 180 equal portions, and divisions equal to these marked off above  $212^{\circ}$  and below  $32^{\circ}$ . In the scale of Celsius, better known as the centigrade scale, the freezing point is marked  $0^{\circ}$ , and the boiling point  $100^{\circ}$ , the tube between these points being divided into 100 equal parts, and equal divisions marked off along the rest of the tube. This scale of temperature is now almost universally adopted by scientific men. The third scale of temperature, viz., that of Réaumur, is in very general use in ordinary life on the Continent. In it, as in the centigrade scale, the freezing point is marked  $0^{\circ}$ , but the boiling point is marked  $80^{\circ}$ . This scale, like that of Fahrenheit, has little to recommend it in preference to the centigrade, except its general use in some localities.

16. We shall now suppose that we are graduating a centigrade thermometer, and that the freezing and boiling points have been marked upon its stem as described above. If the thermometer is not required for very accurate measurements, the remainder of the work is very simple. The length of tube between the freezing and boiling points is divided into 100 equal parts, and these are marked off upon the tube, or on a boxwood or metal scale attached to it, by means of a dividing engine or otherwise, equal divisions being then carried on throughout the remainder of the tube. Each division thus marked is called a degree, and those degrees which lie below the freezing point, or zero of the scale, are marked negative.

### 17. Calibration of the Thermometer Tube.

Now, it is obvious that if there be any inequalities in the bore of the thermometer tube in different parts, equal lengths of the tube will not always correspond to equal increments in the volume of the mercury relative to that of the glass. The above mode of graduation is therefore not sufficiently exact for accurate instruments. In order to remove this cause of error, a small pellet of mercury, of a convenient length, is detached from the rest of the column, and by inclining and gently tapping the tube, is made to occupy different positions within it. The length of the pellet in each position is carefully measured, and, since its volume is constant, it follows that, if the length of a degree at any part of the tube is made proportional to the length of the pellet when occupying this position, each degree will correspond to the same increment of the *apparent* volume of the mercury in the glass. This process is called calibrating the tube. The division of the tube is then conducted on this principle, and it is obvious that care and calculation are required in effecting it. A small pellet of almost any desired length can generally be removed from the rest of the column by tapping it so as to remove a long pellet, warming it, re-uniting the pellet to the column, cooling it, and again tapping the tube. A pellet will then break off at the same point of the tube at which the last united to the column, and the column being shorter in this case, the pellet will be shortened by an equal amount. By carefully repeating this process we can remove a very short pellet.

All thermometers pretending to accuracy should be graduated on the glass tubes themselves, and not on a scale which can be detached from the instrument.

It has been remarked (Art. 9) that the expansion of the mercury with which we have been dealing, is not its actual increase of volume, but its *apparent* expansion when contained in a glass vessel, and is the difference between the actual increase of volume of the mercury and that of the interior of the part of the glass vessel which originally contained it.

18. A thermometer in which the freezing and boiling points have been actually determined as above described, is sometimes called a "natural standard."

Thermometers with capacious bulbs and fine tubes, in which, therefore, a small change of temperature produces a considerable change in the length of the column of mercury, are called "open range thermometers."

Thermometers whose bulbs are very small and thin, and which, therefore, quickly assume the temperature of the body in which they are immersed, are called "sensitive thermometers."

19. **Relations between Thermometric Scales.** It is very important to be able to convert temperatures expressed in one of the three above-mentioned scales into the corresponding temperatures of either of the other scales.

Suppose  $F^\circ$ ,  $C^\circ$ , and  $R^\circ$  to represent the scale-reading of Fahrenheit's, the centigrade, and Réaumur's thermometers respectively, corresponding to any, the same, given temperature. Now, the freezing point on Fahrenheit's scale is marked  $32^\circ$ . Hence,  $F - 32$  is the number of degrees Fahrenheit of the given temperature *above the freezing point*, while  $C$  and  $R$  express the same in degrees of the centigrade and Réaumur's scales respectively. But the excess of the temperature of the boiling point above that of the freezing point is  $180^\circ$ ,  $100^\circ$ , and  $80^\circ$  on the three scales respectively, and the excess of any other temperature above that of the freezing point expressed in degrees of these scales will therefore be proportional to these numbers. Therefore,

$$F - 32 : C : R :: 180 : 100 : 80.$$

We have therefore the following rules:—

(I) Since  $C : R :: 100 : 80 :: 5 : 4$ , in order to convert temperatures on the centigrade scale into corresponding temperatures on Réaumur's scale, multiply by 4 and divide by 5. To perform the inverse process, *i.e.*, to convert degrees Réaumur into degrees centigrade, multiply by 5 and divide by 4.

(II) Since  $F - 32 : C :: 180 : 100 :: 9 : 5$ , in order to convert temperatures expressed in Fahrenheit's scale into the corresponding centigrade temperatures, subtract 32 from the reading of Fahrenheit's scale, and multiply the result by 5 and divide by 9. To perform the inverse process, multiply the reading of the centigrade scale by 9, divide by 5 and add 32.

For example,

$$212^{\circ} \text{ F.} = \frac{5}{9}(212 - 32)^{\circ} \text{ C.} = \frac{5}{9}180^{\circ} \text{ C.} = 100^{\circ} \text{ C.}$$

$$55^{\circ} \text{ C.} = \left(\frac{9}{5}55 + 32\right)^{\circ} \text{ F.} = (99 + 32)^{\circ} \text{ F.} = 131^{\circ} \text{ F.}$$

(III) Since  $F - 32 : R :: 180 : 80 :: 9 : 4$ , in order to convert readings of Fahrenheit's scale into the corresponding readings on Réaumur's scale, and *vice versa*, proceed as in rule (II), only replacing the 5 by 4. Thus:

$$212^{\circ} \text{ F.} = \frac{4}{9}(212 - 32)^{\circ} \text{ R.} = \frac{4}{9}180^{\circ} \text{ R.} = 80^{\circ} \text{ R.}$$

$$44^{\circ} \text{ R.} = \left(\frac{9}{4}44 + 32\right)^{\circ} \text{ F.} = (99 + 32)^{\circ} \text{ F.} = 131^{\circ} \text{ F.}$$

It may be useful to remark that certain temperatures on Fahrenheit's scale, in addition to the freezing and boiling points, have received particular names; for example,  $55^{\circ} \text{ F.}$  is marked *temperate*,  $76^{\circ} \text{ F.}$  *summer-heat*, and  $98.4^{\circ} \text{ F.}$  *blood-heat*.

The student unaccustomed to such calculations will find it useful to work out the following examples.

Express in degrees centigrade the following temperatures:—

1.	$56^{\circ} \text{ R.}$	Ans.	$70^{\circ} \text{ C.}$
2.	$77^{\circ} \text{ F.}$	"	$25^{\circ} \text{ C.}$
3.	$59^{\circ} \text{ F.}$	"	$15^{\circ} \text{ C.}$
4.	$329^{\circ} \text{ F.}$	"	$165^{\circ} \text{ C.}$
5.	$56^{\circ} \text{ F.}$	"	$13.3^{\circ} \text{ C.}$
6.	$98^{\circ} \text{ F.}$	"	$36.6^{\circ} \text{ C.}$
7.	$108^{\circ} \text{ F.}$	"	$42.2^{\circ} \text{ C.}$
8.	$10^{\circ} \text{ F.}$	"	$-12.2^{\circ} \text{ C.}$

Express in degrees Fahrenheit :

1.	56° R.	Ans.	158° F.
2.	40° C.	"	104° F.
3.	60° C.	"	140° F.
4.	-40° C.	"	-40° F.
5.	-10° C.	"	14° F.
6.	309° C.	"	588.2° F.

20. Coloured sulphuric acid is sometimes used instead of mercury for filling thermometers, it having the advantage of expanding more for a given change of temperature, and thus allowing a wider tube to be employed, so that the surface is more distinctly visible. Coloured alcohol is employed for very low temperatures, since mercury solidifies at about  $-39^{\circ}\text{C}$ . Alcohol is also used in *minimum* thermometers, which will be presently described. When liquids other than mercury are employed, the thermometers are generally graduated by comparing them with a standard mercurial thermometer at various temperatures, and sub-dividing the tube between the graduations so obtained, and, in the case of the alcohol thermometer, marking off, on the portion of the tube which corresponds to temperatures below those measurable by the mercurial thermometer, divisions equal to the lowest of those which can be directly determined by the latter instrument.

## 21. Arbitrary character of Thermometric Scales.

It has been stated (Art. 8) that the scale of the mercurial thermometer is a purely arbitrary scale (quite irrespective of the number of degrees into which the interval between the freezing and boiling points is divided), depending on the peculiar properties of mercury and the crown-glass envelope in which it is enclosed. This may be rendered very apparent by filling several thermometers with different liquids, accurately marking off the freezing and boiling points upon each, and dividing the tubes between these points each into 100 equal parts, as in the construction of the ordinary centigrade thermometer. On placing these thermometers in water at various temperatures, it will generally be found that no two of them precisely agree in their indications, except at the freezing

and boiling points. If one of the thermometers contain water, when the mercurial thermometer registers  $4^{\circ}\text{C.}$ , the water thermometer will stand below zero, and when the mercurial thermometer is at  $16^{\circ}$ , the water thermometer will register about  $1^{\circ}$ .

It should be noticed that the discrepancy between these different thermometric scales is not due simply to one liquid expanding more or less than another, but to the different liquids expanding according to *different laws*, so that those intervals of temperature which produce equal increments of volume in one liquid, do not correspond to equal increments in the volume of the other; hence if one liquid be supposed to expand *uniformly* as the temperature rises, the *rate* of expansion of the other liquids will either increase or diminish with increase of temperature, or it may sometimes increase and at other times decrease for the same liquid.

**22. Merits of the Mercurial Thermometer.** Although different liquids indicate different scales of temperature, yet there is a class of bodies, viz., those which have until recently been regarded as permanent gases, which all expand very nearly according to the same law, and therefore indicate nearly the same scale of temperature. This joint testimony in favour of this particular scale affords a good reason for preferring it to any other, and additional reasons will be mentioned hereafter\* which conspire to render this scale peculiarly valuable. Now this scale of temperature, generally known as the scale of the air thermometer, differs very slightly indeed from that indicated by the expansion of mercury in a crown-glass envelope, and this of itself affords sufficient reason why mercury should be preferred to other liquids for the construction of thermometers.

**23.** The other reasons for the employment of mercury in thermometers are:—

\* The scale of the air thermometer possesses also another peculiarity, viz., that the *same amount of heat* is required to raise the temperature of a given mass of air at constant pressure through each degree of the scale, and it nearly agrees with the absolute thermometric scale deduced from certain thermodynamic considerations briefly referred to in Art. 8 and in Chapter X.

(1) It readily transmits heat through its substance (*i.e.*, is a good conductor), and thus quickly takes up the temperature of the body in which it is placed.

(2) Compared with other liquids it requires very little heat to raise its own temperature through a given range (*i.e.*, its specific heat is small), and this not only favours its quickly assuming the temperature of the body surrounding it, but also enables it to come into thermal equilibrium with the surrounding body without changing the temperature of the latter to any great extent.

(3) It remains liquid at ordinary pressure through a very great range of temperature, *viz.*, from  $-39^{\circ}\text{C.}$  to  $350^{\circ}\text{C.}$

(4) It can readily be obtained in a state of purity.

(5) It does not wet the glass envelope in which it is placed.

24. When mercurial thermometers of extreme accuracy are required for scientific purposes, their indications are sometimes compared with those of the air thermometer, and a table of corrections made; so that temperature can be determined by them as nearly as possible in accordance with the scale of the air thermometer. Some of the forms of the last-mentioned instrument will be described in Chapter IV.

Throughout the remainder of this work all temperatures will be expressed in degrees of the mercurial thermometer, except when otherwise stated.

25. **Registering Thermometers.** The *maximum* thermometer is an instrument used for registering the highest temperature to which it has been exposed since it was last adjusted. Rutherford's maximum thermometer consists of an ordinary mercurial thermometer, in the tube of which is placed a small index, or needle, of black glass or enamel, represented by *ab* in Fig. 3, which shows a section of the tube in the neighbourhood of the index. The thermometer is kept horizontal, and when the mercury expands, its convex surface *c* pushes the index before it, for the same reason for which the surface of water is

FIG. 3.



capable of supporting a fine dry needle. When the mercury contracts it leaves the needle behind, as shown in the figure. The end *a* of the needle will therefore always indicate the highest temperature to which the instrument has been exposed since it was last adjusted. To adjust the instrument the index is brought back into contact with the mercury by placing the thermometer in a vertical position.

26. The minimum thermometer is employed to register the lowest temperature to which it has been exposed. Rutherford's minimum thermometer consists of an alcohol thermometer, placed horizontally, and containing in its tube a small index of glass, or enamel, as shown in Fig. 4.

FIG. 4.



Since the alcohol wets the glass, its surface will be concave, as shown at *c*, instead of convex, as in the case of mercury. When the alcohol contracts, the surface *c* pushes the index back with it, but when it expands, the index being entirely wetted by the spirit, it is left behind. The end *a* of the index will then show the lowest temperature to which the thermometer has been exposed. The instrument is set by inclining it until the index slides down to the surface *c* of the spirit.

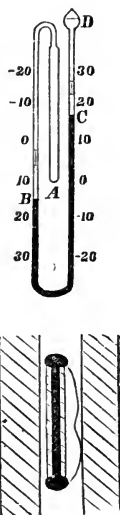
27. In Phillips' maximum thermometer the column of mercury is divided, so that a short thread is separated from the rest of the column by the introduction of a small quantity of air. When in use, the tube is placed in a horizontal position, but in order to set the instrument it is held with the bulb downwards, and gently tapped. By this means the separated thread of mercury is caused to fall towards the rest of the column, and to compress the air between the two until the length of the tube occupied by the air is (in general) not more than one-twentieth of an inch. When the tube is placed in a horizontal position, and the temperature is raised so as to cause the mercury to expand, the pressure of the bubble of air pushes the thread of mercury before it, so that the end of the thread indicates the highest temperature attained; but when the temperature is lowered, and the mercury



contracts, there being no air in the portion of the bulb which is beyond the thread of mercury, there is nothing to force the thread back, and it remains stationary, indicating the highest temperature which the thermometer has experienced since it was last set.

28. Of all self-registering thermometers, perhaps the best known is Six's, which serves at once as a maximum and minimum thermometer. The bulb *A* (Fig. 5) is filled with spirit. The thermometer tube is bent, as shown in the figure, and contains a quantity of mercury which occupies about half the entire length of the tube. Above the surface *C* of the mercury is placed a second quantity of spirit which flows into the bulb *D* when the temperature is sufficiently raised. The only object of this spirit is to facilitate the motion of the index which registers the highest point to which the surface *C* of the mercury has risen. Two scales are provided, one for the surface *B*, and the other for *C*. Each index is made of iron or steel, surrounded by glass and having a fine glass fibre attached, which serves as a spring, and, pressing on the interior of the tube, enables the index to be supported by friction. (See Fig.) When it is required to adjust the instrument for use, the indices are drawn down the tube by a magnet till they are in contact with the mercury. As the mercury moves on account of the expansion or contraction of the alcohol in the bulb *A*, the needle at *C* or *B* is pushed up the tube by the surface of the mercury, as in Rutherford's maximum thermometer. Thus the lower surface of the index in the tube *CD* marks the highest temperature to which the bulb *A* has been exposed, while the index in *AB* registers the lowest temperature. The motion of the surface *C* is due to the expansion of the alcohol together with that of the mercury, so that the degrees of the maximum scale ought to be a little longer than those of the minimum scale; but this

FIG. 5.



difference is scarcely sensible, amounting in the instrument shown in Fig. 5 to about one and a half per cent.

29. The weight-thermometer may frequently be employed as a convenient form of maximum thermometer, but this will be described when we speak of the expansion of liquids (Art. 80).

In Negretti and Zambra's maximum thermometer the tube is bent at right angles just above the bulb, and is choked at the bend so as nearly to divide the column of mercury. When in use, the stem of the instrument is made to slope slightly downwards. As the mercury expands, the thread is pushed past the obstruction; but, on contracting, the mercury is divided at the bend, and the thread left behind in the tube.

Thermometers employed for the measurement of very high temperature are called pyrometers. Some of these instruments will be described later on.

The most delicate instruments for detecting and measuring very small differences of temperature depend on thermo-electric actions, and will be described in Chapter VII.

By causing photographic paper to travel slowly behind the thermometer tube in a direction at right angles to the length of the tube, while light reaches it only through the upper part of the tube above the mercury, a permanent record of the variations of temperature may be obtained.

## CHAPTER II.

ON HEAT CONSIDERED AS A QUANTITY. SPECIFIC HEAT.  
CALORIMETRY. TABLE OF SPECIFIC HEATS.

30. **Physical Quantities.** DEF. *A quantity is that which can be expressed as a multiple, part, or parts of an arbitrary unit of its own kind.*

The complete representation of a physical quantity will therefore consist of two factors: one of which expresses the unit, *which must be of the same kind as the quantity considered*; while the other is a pure number expressing the ratio which the quantity bears to that unit, and is called the measure of the quantity. [From this it will be seen that a quantity can be measured when its ratio to a definite quantity of the same kind can be determined. Now the determination of the ratio which one quantity bears to another, regarded as a process of division, is but the inverse of multiplication, and is possible if the latter operation can be performed upon the quantities. But the multiplication of a quantity by a pure number is possible whenever the operation of adding together two or more quantities of the same kind as that considered is capable of a rational interpretation; and hence it follows that the test of *quantity* properly so called, that is, of a measurable quantity, is the possibility of giving a rational interpretation to the result obtained when the quantity is added to another of the same kind.]

31. In the preceding chapter we have discussed the modes of measuring temperature, and we have throughout spoken of *degrees*, and not *quantities*, of temperature; because hitherto we have seen in temperature only a *quality* which affects bodies, and not a *quantity* which can be

dealt with by the ordinary laws of arithmetic; for in so far as we have at present discussed temperature, we can assign no meaning whatever to the operation of adding together two temperatures, or of multiplying a temperature by any number. If we mix together two equal quantities of a substance at the same temperature, the temperature of the mixture is not the sum of the temperatures, but is *equal to that of either*; and if their temperatures be originally unequal, the temperature of the mixture is intermediate between that of its components. In fact, if we have no other conception of temperature than that adopted in the preceding chapter, the operation of adding together two temperatures is altogether unintelligible. At any rate, it is at once obvious that we must not add the numbers representing the temperatures on any of the arbitrary scales we have described, for the results will be different according to the position of the zero of the scale, though the temperatures are the same. Thus  $15^{\circ}\text{C.}$  is the same temperature as  $59^{\circ}\text{F.}$ , and  $20^{\circ}\text{C.}$  the same as  $68^{\circ}\text{F.}$ ; but  $35^{\circ}\text{C.}$  is not the same as  $127^{\circ}\text{F.}$ , but is equal to  $95^{\circ}\text{F.}$  Temperature is in this respect analogous to many other qualities possessed by bodies; as, for example, hardness. We may say that a body *A* is harder than a second body *B*, if *A* will scratch *B*, but cannot be scratched by it; and we may then select a number of bodies, each of uniform hardness, and number them so as to form an arbitrary scale. The number attached to each substance will then represent its degree of hardness. We may, for example, take the hardnesses of the following substances as the standards of reference for our arbitrary scale, calling that of Talc 1, Rocksalt 2, Calcareous spar 3, Flour spar 4, Apatite 5, Felspar 6, Rock-crystal 7, Topaz 8, Corundum 9, Diamond 10. Now, suppose a piece of glass will scratch Apatite but is scratched by Felspar, we say then that its degree of hardness lies between 5 and 6. Again, suppose a pen-knife will scratch Felspar but not Rock-crystal, then we say that its hardness lies between 6 and 7. We cannot however assign any meaning to the operation of adding together the hardness of the glass and knife, and so can never have to determine whether the result would

be a hardness greater than that of the diamond or of any other substance, nor can we compare the interval between the hardnesses of any two substances in our scale with the corresponding interval for any other pair.

32. The case is however different with the agent which is the cause of increase of temperature in bodies; namely, heat itself. If we take a pound of water at  $1^{\circ}\text{C}$ ., it will require a certain definite amount of heat to raise its temperature to  $2^{\circ}\text{C}$ . under given conditions; and a second pound of water will require an equal amount of heat to produce the same change of temperature under the same circumstances. Consequently the two pounds require *twice* the amount of heat required by the one pound to produce the same change of temperature; so we see that heat must be of the nature of a physical *quantity*, which is subject to the ordinary operations of arithmetic (*i.e.* addition, subtraction, etc.), and we are therefore justified in speaking of *amounts*, and not of *degrees*, of heat.

33. **Unit of Heat.** Since, then, heat is a physical *quantity*, it is of importance to compare the amounts of heat required to produce given changes in definite portions of matter. If a pound of water at  $0^{\circ}\text{C}$ . be mixed with a pound of water at  $2^{\circ}\text{C}$ ., the mixture will be found to have a temperature of  $1^{\circ}\text{C}$ .; so that the heat which leaves 1 lb. of water when its temperature falls from  $2^{\circ}\text{C}$ . to  $1^{\circ}\text{C}$ ., is just capable of raising the temperature of 1 lb. of water from  $0^{\circ}\text{C}$ . to  $1^{\circ}\text{C}$ . We learn, therefore, from this experiment, that the *same amount of heat* is required to raise the temperature of 1 lb. of water from  $1^{\circ}\text{C}$ . to  $2^{\circ}\text{C}$ . as from  $0^{\circ}\text{C}$ . to  $1^{\circ}\text{C}$ ., and therefore the amount of heat required to raise 1 lb. of water from  $0^{\circ}\text{C}$ . to  $2^{\circ}\text{C}$ . is *twice* that required to raise it from  $0^{\circ}\text{C}$ . to  $1^{\circ}\text{C}$ . Similarly, if we mix together 1 lb. of water at  $0^{\circ}\text{C}$ . and 1 lb at  $4^{\circ}\text{C}$ ., the temperature of the mixture will be found to be  $2^{\circ}\text{C}$ ., from which we infer that the amount of heat required to raise 1 lb. of water from  $2^{\circ}\text{C}$ . to  $4^{\circ}\text{C}$ . is equal to that required to raise it from  $0^{\circ}\text{C}$ . to  $2^{\circ}\text{C}$ .; whence it follows that the amount of heat required to raise 1 lb. of water from  $0^{\circ}\text{C}$ . to  $4^{\circ}\text{C}$ . is four times that required to raise it from  $0^{\circ}\text{C}$ . to  $1^{\circ}\text{C}$ . If we proceed in this way with water at higher

temperatures, we shall find that the temperature of the mixture is rather higher than the mean of its components. Thus, if we mix 1 lb. of water at  $0^{\circ}\text{C.}$  with 1 lb. at  $80^{\circ}\text{C.}$ , the temperature of the mixture will be about  $40.14^{\circ}\text{C.}$ ; from which we infer that the amount of heat required to raise a pound of water  $1^{\circ}\text{C.}$  when at a high temperature, is rather greater than that required to raise it  $1^{\circ}\text{C.}$  at a low temperature. If, however, the temperature of the water be not high, we may say that the amount of heat required to raise the temperature of a pound of water through  $N^{\circ}\text{C.}$  is  $N$  times the amount required to raise the same quantity of water from  $0^{\circ}\text{C.}$  to  $1^{\circ}\text{C.}$

34. As stated in Art. 30, the expression of any physical quantity consists of two factors: one of which is the unit in terms of which it is measured, and *of the same nature as the quantity considered*; while the other is a pure number, showing how many times the quantity contains this unit, and is called the *measure* of the quantity.

Hence, in order to express the magnitude of any physical quantity, we must fix upon an invariable unit of its own kind.

DEF. *The unit of heat is the amount of heat required to raise the temperature of 1 lb. of water from  $0^{\circ}\text{C.}$  to  $1^{\circ}\text{C.}$*  This quantity of heat is sometimes called a *calorie*, though this term is more properly confined to the amount of heat required to raise a gramme of water through the same range of temperature.

In nearly all formulæ pertaining to physical quantities, it is the measures alone which are represented by symbols, the units being either supposed to be understood or else expressed in words. Hence these symbols may be treated as pure numbers. Since, however, the measures of quantities change when the units are altered, in order that an equation may be true independently of the particular units adopted, the quantities represented on the two sides of an equation must be of the same nature.

35. **Specific Heat.** DEF. *The specific heat of a substance at any temperature is the ratio of the amount of heat*

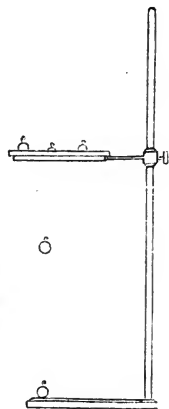
required to raise the temperature of any mass of the substance  $1^{\circ}\text{C}$ . from the given temperature, to the amount of heat required to raise the temperature of an equal mass of water from  $0^{\circ}\text{C}$ . to  $1^{\circ}\text{C}$ .

From this definition it will be seen that the specific heat of a substance at any temperature is equal to the number of *units of heat* required to raise the temperature of one pound of the substance  $1^{\circ}\text{C}$ . from that temperature.

*The number of units of heat required to raise the temperature of a body  $1^{\circ}\text{C}$ . is called its CAPACITY FOR HEAT.*

36. From these definitions it will be seen that the *specific heat* of a body depends only on the kind of matter of which it is composed, and the physical conditions under which it is placed, while the *capacity for heat* of a body is also proportional to the amount of matter it contains. The fact that equal volumes of different metals have different capacities for heat may be shown experimentally by heating a number of equal spherical balls of different metals, say copper, iron, tin, zinc, lead and bismuth, to  $100^{\circ}\text{C}$ ., and then laying them on a thin cake of wax. The copper ball will quickly melt its way through the wax, and be followed almost immediately by the iron ball; the copper preceding the iron because, though its specific heat is only  $\cdot 095$ , while that of iron is  $\cdot 113$  (per unit of mass), yet the density of the copper is greater than that of the iron, and the copper also more readily allows of the passage of heat from its interior. Shortly after the iron, the zinc ball, whose specific heat is  $\cdot 0955$ , will fall, while the lead, whose specific heat is  $\cdot 03$ , and the tin, whose specific heat is  $\cdot 056$ , will perhaps be unable to melt their way through, and the bismuth ball, whose specific heat is nearly the same as that of lead, but whose density is much less, will only make a comparatively shallow impression in the wax.

FIG. 6.



The metal balls are heated together in an oil-bath, all

of them being hung from a small wire frame, by means of which they are simultaneously removed and placed on the cake of wax.

37. Referring to the properties of water mentioned in Art. 33, it will be seen that the specific heat of water is nearly unity, if its temperature be not much above  $0^{\circ}\text{C.}$ , but at higher temperatures its specific heat is rather greater than unity. It continues to increase with the temperature. (Art. 52.)

38. **Measurement of Heat.** The operation of measuring quantities of heat is called *calorimetry*; and an instrument adapted to this purpose is called a *calorimeter*.

The process of determining the specific heat of a substance consists in finding the amount of heat required to produce a given change of temperature in a known mass of the substance, and is therefore an operation in calorimetry. We shall now describe some of the methods usually adopted for measuring quantities of heat.

#### THE METHOD OF MIXTURE.

39. An illustration of this method has already been given, when we showed how to prove experimentally that the amount of heat required to raise the temperature of a pound of water from  $1^{\circ}\text{C.}$  to  $2^{\circ}\text{C.}$  was equal to the amount required to raise an equal mass of water from  $0^{\circ}\text{C.}$  to  $1^{\circ}\text{C.}$  It consists essentially in mixing together known quantities of two substances at different temperatures, the specific heat of one of which is known, and determining the temperature of the mixture. In most cases water is taken as the substance whose specific heat is known; the other substance may be a liquid or a solid, but it must not act chemically on the water. The method will be best understood by taking another example.

40. Suppose we wish to determine the specific heat of copper. We weigh out a convenient quantity, say 50 grains, of copper, and suspend it by a fine thread in the steam above the surface of water boiling in a vessel almost closed, but allowing sufficient means of egress for the steam to prevent its pressure becoming sensibly greater than that of the atmosphere. If the barometer stand at



30 inches, the copper will after some time have acquired a temperature of about  $100^{\circ}\text{C}$ . While the copper is being heated, we weigh out carefully a quantity of distilled water, say 1000 grains. This is placed in a vessel whose capacity for heat is small, and after the vessel and water have acquired the same temperature, this temperature is very carefully observed. Suppose it to be  $12^{\circ}\text{C}$ . The copper is now removed from its hot bath and plunged as quickly as possible into the cold water, where it is moved up and down by means of its suspension thread, until thermal equilibrium between it and the water is attained. (It is important that this process should require as short a time as possible, and to this end the copper should not have a very small surface. A coil of copper wire answers the purpose very well.) The temperature of the water is then again carefully observed; and the time elapsing between the immersion of the copper and the observation of this temperature should also be noticed. Suppose the temperature to be  $15.7^{\circ}\text{C}$ . The ratio of the mass of the copper to that of the water in the calorimeter should, if possible, be so arranged that the temperature of the air in the room is intermediate between the first and last observed temperatures in the calorimeter, and somewhat nearer to the latter than the former. This can be insured if the specific heat of copper be known approximately. The loss of heat by radiation and conduction from the instrument will then be corrected by a nearly equivalent gain, but in extremely accurate measurements this must be determined and allowed for. This condition would be fulfilled in the case we are considering, if the temperature of the air in the room were  $14^{\circ}\text{C}$ . or  $15^{\circ}\text{C}$ ., or even somewhat higher. Now, if we neglect the capacity for heat of the vessel containing the water, we have sufficient data to obtain a rough approximation to the specific heat of copper. The number of degrees of temperature lost by the 500 grains of copper is  $100 - 15.7$ , or  $84.3$ ; and the heat leaving the copper serves to heat 1000 grains of water  $3.7^{\circ}\text{C}$ ., and would therefore heat 500 grains of water  $7.4^{\circ}\text{C}$ . The mean specific heat of copper between the temperatures  $15.7^{\circ}\text{C}$ . and  $100^{\circ}\text{C}$ . is therefore

$$\frac{7.4}{84.3} = .0877 \dots \text{according to this measurement.}$$

41. The result just obtained is, obviously, too small, because we have neglected the heat taken up by the vessel containing the water, when the temperature of the latter is raised. In order to determine this approximately, we may refill the vessel with cold water, and after allowing it to stand for a considerable time, observe its temperature. Suppose it again to be  $12^{\circ}\text{C}$ . Now empty the vessel as thoroughly as possible, and place in it 1000 grains of water at, say,  $15^{\circ}\text{C}$ ., this temperature being accurately observed before pouring the water into the vessel. After this has stood for some time, say rather less than the time above referred to, let the temperature be again observed. Suppose it to be  $14.77^{\circ}\text{C}$ . Now the amount of water at  $12^{\circ}\text{C}$ ., which must be mixed with 1000 grains at  $15^{\circ}\text{C}$ . to reduce the whole to  $14.77^{\circ}\text{C}$ . is about 83 grains. The presence of the vessel then is equivalent to increasing the amount of water from 1000 to 1083 grains. Making allowance for this, we see from the results just obtained, that the amount of heat required to raise the temperature of 500 grains of copper  $84.3$  degrees will raise 1083 grains of water  $3.7$  degrees; and will therefore raise the temperature of 500 grains of water

$$\left(\frac{3.7 \times 1083}{500}\right)^{\circ}\text{C}.$$

The mean specific heat of copper between  $15.7^{\circ}\text{C}$ . and  $100^{\circ}\text{C}$ . is therefore

$$\frac{3.7 \times 1083}{500 \times 84.3} = .095...$$

This result is very nearly correct.

As another example of this method, the student may take the following:—

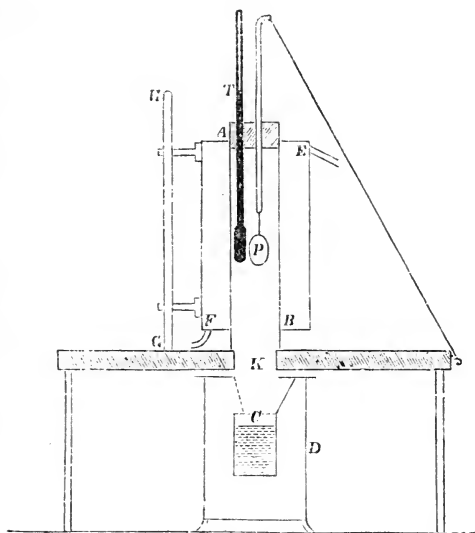
*1000 grammes of mercury at  $100^{\circ}\text{C}$ . are mixed with 1000 grammes of water at  $15^{\circ}\text{C}$ . contained in a vessel which, on account of its capacity for heat, is equivalent to an increase of 100 grammes in the quantity of water within it. The resulting temperature is  $17.5^{\circ}\text{C}$ . Find the specific heat of mercury.*

Ans. .033.

42. Notwithstanding the precautions we have mentioned and the corrections we have applied, there are

still some sources of error incidental to the above method which continue to vitiate our results. Two notable sources of error are (1) the condensed steam which clings to the substance and is conveyed with it into the calorimeter, thus increasing the apparent value of the specific heat, and (2) the loss of heat while passing from the heater into the calorimeter. Both these sources of error are eliminated in the following method, which will be found to give tolerably accurate results. The apparatus

FIG. 7.



is represented in Fig. 7. The calorimeter, *C*, is a cylindrical vessel made of thin copper and polished or electroplated on the outside, in order to diminish radiation from the surface. This is suspended by silk strings within a second copper vessel, *D*, which is polished or plated on the inside, so as to reflect any radiation it may receive from the calorimeter. This vessel is closed by a lid having in its centre a hole sufficiently large to allow the body whose specific heat is to be measured to pass through it into the calorimeter. By this means the loss of heat

from the calorimeter is reduced to a minimum, while the temperature of the copper vessel, *C*, never differs sensibly from that of the water within it, and the heat absorbed by it can therefore be accurately determined and added to that absorbed by the water. The heater, which is a modification of that of Regnault, consists of a thin copper tube *AB*, surrounded by a large tube *EF*, the space between them being closed at the top and bottom, but communicating at *E* with a steam boiler, while at *F* there is a tube for the escape of the steam. The whole apparatus rests upon a board, and is capable of turning about the vertical pillar *GH*, which serves as a guide, when it is required to bring the opening of the tube vertically over the hole *K* in the board, in order to drop the substance into the calorimeter. For this purpose the latter is quickly brought under the hole *K* and removed again as soon as possible, the board in the meanwhile serving to screen the calorimeter from radiation from the steam-jacket. The tube *AB* is open at the bottom, the board itself serving to close it except when it is brought over the hole *K*; but the top is closed by a cord, through which passes a thermometer *T*, and a glass tube of very fine bore, which admits the passage of a fine silk thread by which the body, *P*, to be heated is suspended. When the steam has been turned on, it should be allowed to blow through the steam-jacket for some time after the thermometer has become perfectly steady, in order to ensure the body *P* having the same temperature as the thermometer throughout its substance. The temperature of the water in *C* having been observed, the calorimeter is quickly brought beneath the hole *K*, while the tube *AB* is brought over it, and the body *P* lowered into the water by means of the thread. The calorimeter is then removed and the measurement completed as in the case above described.

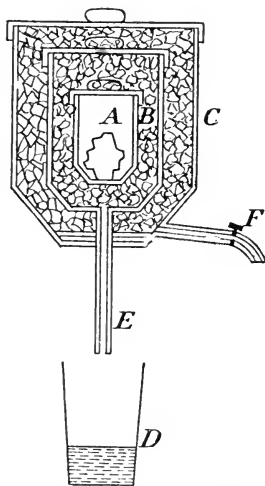
It is worthy of notice that the thermometer and body to be heated will never quite attain the temperature of the steam, because they are not completely surrounded by the steam-jacket. The inner chamber in which they are placed is closed above by the india-rubber cork and below by the board, and neither of these substances attains the

temperature of the steam. Hence the thermometer and the suspended body will lose heat by radiation to them, and will thus always be at a temperature below that of the surrounding steam.

#### THE METHOD OF FUSION.

43. If we mix together 1 lb. of water at  $79^{\circ}\text{C}$ . with 1 lb. of water at  $0^{\circ}\text{C}$ ., we obtain 2 lbs. of water at very nearly  $39.5^{\circ}\text{C}$ . If, however, we mix together 1 lb. of water at  $79^{\circ}\text{C}$ . with 1 lb. of ice at  $0^{\circ}\text{C}$ ., we obtain 2 lbs. of water at  $0^{\circ}\text{C}$ . From this we see that in order to melt 1 lb. of ice at  $0^{\circ}\text{C}$ ., without changing its temperature, we require as much heat as would raise 1 lb. of water  $79^{\circ}\text{C}$ .; in other words, we require 79 units of heat. This number, 79, is called the *latent heat* of water, or the latent heat of fusion of ice, and will be more fully discussed in a subsequent chapter; it is introduced here because this property of ice is made the basis of the construction of several instruments for measuring quantities of heat, known as *ice calorimeters*, two of which we shall now describe. We must so far trespass upon the matter of subsequent chapters as to state, that while pure ice is melting under ordinary pressure, the temperature of the ice and water remains accurately at  $0^{\circ}\text{C}$ . till the whole of the ice is melted, a fact of which we have already availed ourselves in determining the freezing point.

FIG. 8.



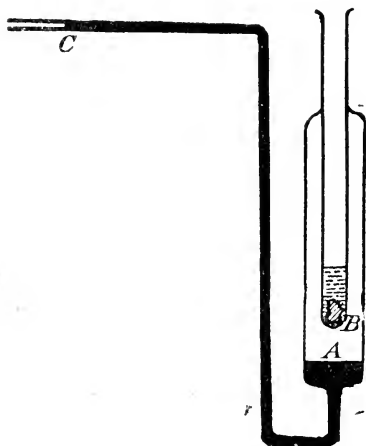
44. The calorimeter of Laplace and Lavoisier consists of three vessels. The inner vessel *A* is formed of thin copper and contains the heated body. This is placed in the interior of a larger vessel, *B*, which forms the calorimeter proper, the space between them being filled with broken ice, and this again is placed within a still larger vessel, also packed with broken ice to prevent absorption of heat from surrounding

bodies by the calorimeter proper. A small pipe passes through the outer vessel from the bottom of the calorimeter proper, and serves to convey the water from *B* into a vessel placed for its reception. Now since *B* is surrounded by ice at  $0^{\circ}\text{C}$ ., the ice within it can only melt by taking up the heat which passes from the body in *A*; and since 1 lb. of ice requires 79 units of heat to melt it, we can determine the amount of heat which leaves the body in *A*, by weighing the water which falls into the vessel *D*. The body in *A* is, of course, finally reduced to the temperature of the ice in *B*; that is, to  $0^{\circ}\text{C}$ . The pipe *F* simply conveys away the waste water from the ice jacket produced by the heat absorbed from surrounding bodies.

45. Suppose the body in *A* to be 500 grains of copper originally at  $100^{\circ}\text{C}$ ., and suppose that the flow of water from *B* ceases when 60 grains have escaped. We know that the temperature of the copper is then  $0^{\circ}\text{C}$ . Now the heat required to convert 60 grains of ice at  $0^{\circ}\text{C}$ . into water at the same temperature would raise  $79 \times 60$  grains, or 4740 grains, of water  $1^{\circ}\text{C}$ . Hence the heat lost by 500 grains of copper in cooling from  $100^{\circ}\text{C}$ . to  $0^{\circ}\text{C}$ . would raise 4740 grains of water  $1^{\circ}\text{C}$ . The mean specific heat of copper between  $0^{\circ}\text{C}$ . and  $100^{\circ}\text{C}$ . is therefore

$$\frac{4740}{500 \times 100} = .0948.$$

FIG. 9.



46. The action of Bunsen's calorimeter depends on the fact that ice contracts considerably when melting, and the water continues to diminish in volume as its temperature increases between  $0^{\circ}\text{C}$ . and  $4^{\circ}\text{C}$ . It consists of a vessel, *A*, containing water, into the upper part of which a test-tube, *B*, is sealed, and the whole vessel is thus closed ex-

cept for the fine tube, *C*, which is graduated, and, together with the lower part of *A*, filled with mercury. When the instrument is to be used, it is immersed in melting snow or ice till the temperature of the water has fallen to  $0^{\circ}\text{C}.$ , when a quantity of alcohol, cooled considerably below the freezing-point by a mixture of ice and salt, is made to circulate through the test-tube. This causes a layer of ice to be formed round the test-tube, while the mercury is forced along the tube *C* on account of the expansion of the water in freezing. Sometimes ether is placed in the test-tube and a current of cold air blown through it, which causes evaporation to take place so rapidly that the temperature falls below the freezing-point. The alcohol or ether is then removed, and cold water is placed in the test-tube, and when it has attained a temperature of  $0^{\circ}\text{C}.$ , as indicated by the surface of the mercury in the tube *C* becoming stationary, the position of the surface in *C* is observed. The body whose capacity for heat is to be measured, having been heated to  $100^{\circ}\text{C}.$  in steam, is now placed in the test-tube *B*. This heats the water in *B*, which in its turn communicates the heat to that in *A*, thus melting some of the ice and causing the surface of the mercury in *C* to move. The decrease in the volume of the contents of *A* is measured by the movement of the mercury in *C*, and thus the amount of ice melted in *A* can be found. Also no heat can have entered *A* except from the test-tube, since it is surrounded by melting snow, and nearly all the heat lost by the body in *B* will enter *A*, for most of the water in *B* being at  $0^{\circ}\text{C}.$ , since water contracts in volume as its temperature rises from  $0^{\circ}\text{C}.$  to  $4^{\circ}\text{C}.$ , that warmed by the hot body is *heavier than the rest*, and does not rise to the surface. The amount of ice melted in *A* therefore enables us to measure the heat lost by the body in *B* in cooling from  $100^{\circ}\text{C}.$  to  $0^{\circ}\text{C}.$  Several measurements can be made with the instrument without again freezing the water which surrounds the tube.

If the area of the cross section of the tube *C* is accurately known, the amount of heat communicated to the contents of the calorimeter can be at once determined by measuring the distance through which the surface of the

mercury in the tube recedes. Thus a gramme of ice occupies about 1090 cubic millimètres, and contracts on melting into little more than 1000 cubic millimètres of water. Hence a diminution of volume of 90 millimètres corresponds to the melting of one gramme of ice or to the acquisition of  $79\cdot25$  gramme-centigrade units of heat. If the section of the tube be  $a$  square millimètres, and the surface of the mercury recede through  $l$  millimètres, this will correspond to the addition of  $\frac{la}{90}$   $79\cdot25$  gramme-centigrade units of heat to the calorimeter. In British units a contraction of a cubic inch corresponds to nearly  $32$  pound-centigrade units of heat.

Instead of determining the section of the tube  $C$ , it is very usual to calibrate it directly so as to correspond to heat units. This may be conveniently done by introducing into the test-tube a small body whose capacity for heat is accurately known and whose temperature has been carefully determined. Thus 14 grains of copper heated to  $15^{\circ}\text{C}$ ., and dropped into the test-tube, would give almost exactly 20 units of heat, and the copper might be removed and re-heated to  $15^{\circ}\text{C}$ . any number of times, until the whole length of the tube  $C$  had been divided into sections each corresponding to 20 grain-centigrade units of heat. At a temperature of  $15^{\circ}\text{C}$ . there would be no necessary loss of heat by the copper in being transferred to the calorimeter.

47. An improved method of employing Bunsen's calorimeter has recently been suggested. Instead of employing a long tube calibrated throughout its length, the tube  $C$  is bent over and the end rounded off, while the bore is made to terminate in a very fine aperture. The end of the tube, which is completely filled with mercury, is made to dip into a small cup of mercury, which is weighed before and after each experiment, and as the ice melts, more mercury enters the calorimeter through the aperture at the end of the tube. The amount of mercury thus entering is determined by the loss of weight of the cup of mercury. Every grain of mercury entering the instrument corresponds to nearly  $65$  grain-centigrade units of heat.



48. **Regnault's Calorimeters.** Other forms of calorimeters have been employed for the measurement of the specific heats of gases and liquids. The most perfect are those devised by Regnault. In order to determine the specific heat of a gas at constant pressure, he made a known quantity pass from a holder through a metal "worm" which was immersed in heated oil. In this it acquired the temperature of the oil, and thence it passed through a complicated metal vessel immersed in the water of the calorimeter. Here it acquired the same temperature as the water, while the temperature of the latter was gradually raised. The temperature of the water being observed at the commencement and close of the experiment, after making proper correction for the capacity of the apparatus, the number of units of heat given up by the gas to the calorimeter could be found. Now all the gas was heated to the temperature of the oil, while the temperature to which it was cooled in the calorimeter was variable. Since, however, the temperature of the calorimeter must have increased almost uniformly, the result was the same as if the whole of the gas had been cooled to the temperature which is the arithmetical mean of the initial and final temperatures of the calorimeter. Knowing the mass of the gas, we have now sufficient data for the calculation of its specific heat.

49. The principal peculiarity of Regnault's calorimeter for liquids is the artifice for preventing loss of heat by the liquid on account of exposure in passing from the bath in which it is heated to the calorimeter. This consists in forcing the liquid, by means of atmospheric pressure, from the vessel in which it is heated, and which is immersed in a hot bath, through a tube into a vessel immersed in the water of the calorimeter, the latter instrument being prevented from receiving heat directly from the hot bath, to which it is placed very close, by means of a screen.

We may observe that in the case of substances which, like water, are known in the solid, liquid, and gaseous states, the specific heat of the liquid is greater than that of either the solid or the gas, provided the volume of the gas be unchanged.

**50. Method of Cooling.** The method of cooling is chiefly valuable for determining the specific heats of liquids. The apparatus required is two copper vessels, very similar to the calorimeter, with its surrounding vessel, described in Art. 42; only in this case the outside of the inner vessel and the inside of the outer vessel are coated with lamp-black, to make the one radiate and the other absorb radiation as readily as possible. The liquid whose specific heat is to be determined is heated and placed in the inner vessel; the aperture in the lid of the outer vessel is closed by a cork through which passes a thermometer, whose bulb dips into the liquid in the inner vessel, and a stirrer by which the liquid is stirred, and thus maintained at the same temperature throughout. The outer vessel is immersed in a large bath of water at the temperature of the room. The time taken by the liquid in the inner vessel to cool through each successive degree is then observed and noted. After the cooling has been continued through a sufficient number of degrees to make it possible to practically eliminate errors of observation, the liquid is removed, and water at the same temperature as the liquid at first possessed is poured into the inner vessel, care being taken that the vessel is filled by the water to exactly the same height as by the other liquid. This is easily secured if the specific gravity of the other liquid be known, as it is then easy to estimate the weight of water required. The time taken by the water to cool through each successive degree is then observed, and compared with the time taken by the other liquid in cooling through the *same* degree on the thermometer scale. The ratio of these two times is taken, and should be the *same for each degree*. Errors of observation will, however, cause the ratios to be slightly different, and the mean of all should be taken as the true ratio. Since the rate at which heat will be lost by the lamp-black surface is always the same at the same temperature (Chap. IX.), it follows that the heat lost by the copper vessel and its contained liquid in cooling through a particular degree is proportional to the time occupied in the cooling. Hence the ratio of the capacity for heat of the copper vessel and the former liquid to the thermal capacity of the same

vessel and the water is known from the experiment. - Also the water equivalent of the copper vessel is known. Hence the thermal capacity of the liquid is at once determined.

Suppose, for instance, that the water equivalent of the copper vessel is 100 grains, and that it contains in the first instance 1840 grains of concentrated sulphuric acid, (which must not be too highly heated in contact with copper), and suppose that the time taken to cool one degree, when the acid is replaced by an equal volume of water (1000 grains), is 1.5 times as long as in the case of the acid. The thermal capacity of the vessel and water is equivalent to that of 1100 grains of water; the thermal capacity of the vessel and acid is two-thirds of this, and is therefore equivalent to that of 733 grains of water. Of this the water equivalent of the copper is 100 grains; therefore the water equivalent of 1840 grains of sulphuric acid is 633 grains, and the specific heat of the acid is therefore .344. A platinum vessel covered with lamp-black would be preferable to a copper vessel for this purpose.

51. The following table gives the specific heat of some well-known substances. The numbers given of course represent the mean specific heats of the various bodies between certain limits of temperature employed in the experiments. These temperatures are given in the third column.

TABLE OF SPECIFIC HEATS OF METALS, ETC.  
(REGNAULT).

				Centigrade.
Bismuth	...	...	.03084	98°—13°
Lead	...	...	.03140	98°—15°
Platinum	...	...	.03243	99°—12°
Gold	...	...	.03244	98°—12°
Mercury	...	...	.03332	98°—12°
Antimony	...	...	.05077	97°—12°
Tin	...	...	.05623	99°—12°
Silver	...	...	.05701	99°—13°
Copper	...	...	.09515	98°—15°
Zinc	...	...	.09555	99°—14°

			Centigrade.
Iron	...	·11380	98°—17°
Sulphur (native)	...	·17760	99°—14°
„ (recently melted)		·20259	98°—14°
Phosphorus	...	·18870	30°—10°
Graphite	...	·20083	98°—12°
Aluminium	...	·21430	97°—14°
Magnesium	...	·24990	98°—23°
Lithium	...	·94080	100°—27°
Brass	...	·09391	98°—12°
Glass	...	·18768	99°—14°

TABLE OF SPECIFIC HEATS OF LIQUIDS.

		Centigrade.
Bromine	·107	45°—11°
Sulphuric acid	·343	46°—21°
Ether	·503	.....
Alcohol	·615	43°—23°

SPECIFIC HEATS OF GASES (AT CONSTANT PRESSURE).

Air	·2374	Sulphurous anhydride	·1544
Oxygen	·2175	Carbonic anhydride	·2163
Nitrogen	·2438	Carbonic oxide	·2450
Hydrogen	3·4090	Nitrous oxide	·2438
Chlorine	·1210	Steam	·4805

In the case of the more perfect gases, the specific heat at constant pressure varies inversely as the density; so that the capacity for heat per unit volume is nearly the same for all gases.

52. As a general rule the specific heats of liquids, as stated in Art. 50, are greater than those of either gases or solids, and the specific heat increases with the temperature. Thus the specific heat of water at 0° C. being taken as unity, its specific heat at 50° C. is 1·0042; at 100° C., 1·013; and at 200° C., 1·044. The specific heat of ice at temperatures near the freezing-point is about ·504.

For full practical details on calorimetry, the student should consult Glazebrook and Shaw's *Practical Physics*.

53. The methods described in this chapter only enable us to measure quantities of heat which pass from one body to another, the receiving body being the water, or ice, of the calorimeter; they will not enable us to form any idea of the whole amount of heat contained in a body, because we cannot get the whole of the heat to pass out of any body into another, and if we could the amount would depend upon the manner in which the heat is abstracted; whether, for instance, the body is allowed to expand and do work, or whether it contracts and has work done upon it by some agent external to itself. Consequently, if we speak of the whole amount of heat contained in a body, we use a phrase to which we can assign no meaning, and, as the student will learn after studying the effects of heat upon gases, the amount of heat required by a body to cause it to pass from one given temperature to another, frequently varies considerably with the conditions as to pressure, etc., to which it is subject, and is not a definite quantity unless these conditions be specified, though its variations are usually extremely small in such cases of liquids and solids as we have generally to deal with. It was on account of these considerations, that in first introducing heat as a measurable quantity, we did not say that the whole amount of heat contained in two pounds of water at  $1^{\circ}\text{C.}$  was equal to twice the amount contained in one pound at the same temperature, but spoke of the amount of heat required to raise different quantities of water from  $1^{\circ}\text{C.}$  to  $2^{\circ}\text{C.}$  *under given conditions.* Since, in the cases of water and most other liquids and solids, the amount of heat required to produce a given change of temperature varies very slightly with ordinary variations of external circumstances, the words in italics are generally omitted; but this statement will not apply to gases.

## CHAPTER III.

### SOURCES OF HEAT.

54. We have seen that one effect of heat upon bodies is, in general, to increase their temperature, and in Chapter I. we have seen how to measure temperature in degrees of an arbitrary scale, while in the last chapter we discussed the measurement of quantities of heat. We must now briefly enumerate the most frequent sources of heat with which we are acquainted, and then consider the effects of heat upon bodies.

55. The principal sources of heat are,—

(I) *Radiant energy.*

(II) *Chemical action (including combustion).*

(III) *Mechanical sources*, such as the energy of a mechanical system converted into heat by doing work against friction, viscosity, pressure (as in cases of impact), etc., or by otherwise producing any change in the form or volume of an imperfectly elastic body.

(IV) *Electric currents*, including those currents which are of very short duration, and which are more generally known as electric discharges, such as lightning flashes.

(V) *The magnetization and demagnetization of iron or other magnetic substances.*

(VI) *Change of state*, such as the condensation of vapours, solidification of liquids, crystallization, or the passage from one crystalline form to another.

(VII) *The internal heat of the earth* differs from the preceding sources, inasmuch as the heat exists in the earth *as heat* before it comes under our notice, and is therefore not a source from which heat is *produced*, but simply a store from which it is *derived*. Volcanic heat has been applied to commercial purposes in the preparation of borax.

56. (I) Of the above sources of heat the radiation from the sun is by far the most important. Radiant energy produces different effects according to the nature of the body which receives it. If it fall upon sensitized photographic paper or plates, it decomposes the silver salt; if it fall upon our eyes, it in some cases produces the sensation of light; but by far the most common effect is the production of heat, an effect of which we are at once conscious if we stand in the sunshine, or in front of a fire. The nature and laws of radiation will be discussed more at length hereafter.

57. (II) Chemical action is generally accompanied by the production or the absorption of heat. When light is also produced, the action is frequently called combustion. The amount of heat produced depends only on the quantities, character, and conditions of the reagents at the beginning and end of the operation, if no energy is abstracted from, or given to, the substances by other systems. Thus a certain amount of heat will be produced by the combustion of two pounds of hydrogen in oxygen, forming 18 pounds of water; and if 39 pounds of potassium be brought into contact with the water so formed, one pound of hydrogen will be liberated, and 56 pounds of caustic potash will be produced, while a certain additional amount of heat will be generated; but the whole quantity of heat produced in the two operations is the same as if 39 pounds of potassium and one pound of hydrogen had been allowed to combine directly with 16 pounds of oxygen and with each other. But while the whole amount of heat which can be obtained from any chemical action, when the acting substances are cooled down to their original temperature, depends only on the quantity of the substances combining, and on the character of the action, the temperature produced depends upon the initial temperatures of the substances, and on the capacity for heat of the products among which the heat is distributed. For example, if hydrogen burn in air, the heat generated is divided between the aqueous vapour produced and the nitrogen of the air; but if it burn in pure oxygen, the whole of the heat is at first confined to

the aqueous vapour produced; and hence the oxy-hydrogen flame is capable of readily fusing platinum, and of producing a much higher temperature than can be produced by hydrogen burning in air.

**58. Regenerator Furnaces.** The influence of the initial temperature of the combining substances on the temperature produced is seen in the effects of the hot blast as applied to blow-pipes and blast furnaces, and to the Siemens furnace. In the last-mentioned furnace the products of combustion are conducted into a chamber partly filled with fire bricks, between which they have to pass before escaping up the shaft, and in this manner the bricks are heated to redness. The draught is then turned aside into a second similar chamber, while the air supplied to the furnace is made to pass through the heated chamber, and when the second chamber has become sufficiently heated, the draught is again reversed, so that all the air supplied to the furnace is raised to a high temperature without employing any other than the waste heat of the furnace itself. This furnace has been employed in the manufacture of steel by the Siemens-Martin process, which consists in fusing wrought iron with such a quantity of cast iron as shall provide the requisite amount of carbon for the steel.

This same principle of the *regenerator* has been applied by Sir Wm. Siemens to gas-lamps for street lighting and other purposes. In these lamps the air, before reaching the gas flame, is made to pass through tubes heated by the flame itself, and thus the temperature of the flame is raised, while the intensity of the light is greatly increased and its tint rendered more white.

**59. Temperature of Ignition.** In order to cause two substances to burn together, it is generally necessary to raise them above a certain temperature. If the temperature produced by their combination be sufficiently high to inflame them, combustion will continue, but otherwise it will cease. The temperature produced by ammonia gas burning in air is insufficient to ignite it, so that the combustion cannot be maintained without an external supply of heat, but in pure oxygen ammonia



will continue to burn. The heat of animals is due to the combination of their food with the oxygen of the air.

**60. Heat developed by Chemical Rearrangements.** In many cases large quantities of heat are produced by a new chemical or molecular arrangement of the constituents of a compound, which is frequently highly explosive. Thus, in nitroglycerine, and some other similar compounds, the oxygen of the nitric acid readily leaves the nitrogen, and combines with the carbon and hydrogen; and as every atom of carbon and hydrogen in the compound has the oxygen requisite for its combustion in close proximity, since the whole forms a homogeneous chemical compound, it is possible for the transformation to take place throughout a considerable mass of the material in an excessively short time, and large volumes of gas being thus almost instantaneously produced at a very high temperature, the explosion is proportionately violent. In the case of gunpowder, the sulphur, carbon, and nitre are only mechanically mixed, so that the carbon and sulphur have some distance to go for the oxygen, and the powder can therefore only burn slowly, and cannot *detonate*.

In other cases a physical rather than a chemical rearrangement of molecules takes place with the production of heat. Some examples of this will be cited hereafter; such, for example, as the crystallization of salts from their solutions, and the sudden passage of red phosphorus into the yellow variety if too highly heated.

Occasionally great heat is produced by the simple decomposition of a compound, the constituents of which have no affinity for one another under the conditions to which they are exposed at the time, having been made to unite under very different circumstances. Nitrogen trichloride, and triiodide, and some of the compounds of chlorine and oxygen, are illustrations of compounds which evolve great heat by simple decomposition, and which decompose with explosive violence.

**61. Calorimetry.** Many calorimeters have been devised for the purpose of measuring the heat produced by chemical action, especially by the combustion of

different fuels, in which case the measurement is of considerable commercial importance. In most of these instruments the products of combustion are made to pass through a long pipe immersed in water in the calorimeter, and the change of temperature of the water is observed. The heating effect of a fuel may be expressed by the number of units of heat produced by the combustion of a pound of the fuel. It is, however, frequently stated in terms of the number of pounds of water at  $100^{\circ}\text{C}$ . which can be converted into steam at the same temperature by the combustion of one pound of the fuel; and, as will be stated more fully hereafter, to convert a pound of water at  $100^{\circ}\text{C}$ . into steam at the same temperature, requires as much heat as would raise the temperature of about 536 pounds of water from  $0^{\circ}\text{C}$ . to  $1^{\circ}\text{C}$ .

62. In the calorimeter devised by Mr. Lewis Thompson of Newcastle-upon-Tyne, which is sometimes employed for comparing the calorific powers of different varieties of coal, the coal is finely powdered, mixed with about fifteen times its weight of a mixture of nitre and potassic chlorate, placed in a crucible within a diving bell, and ignited by a slow match after lowering the diving bell into a vessel containing a known quantity of water. The products of combustion, escaping from the bottom of the bell, bubble up through the water, and are thereby cooled nearly to the temperature of the water itself, which temperature is observed before and after the combustion of the coal. The heat absorbed by the vessel, the diving bell, etc., is estimated and added to that absorbed by the water; the total representing the heat developed by the combustion of the coal employed. With this apparatus it is difficult to secure the complete combustion of the coal; but though it may not be trusted for exact determinations of the absolute amount of heat developed by the coal, it is sufficiently accurate for the *comparison* of the values of different kinds of coal as fuel.

63. In the experiments of Favre and Silbermann, the combustion was carried on in a chamber made of gilded copper plate, and the oxygen, which had been previously purified, was supplied through a tube, while the products

of combustion escaped through a long narrow tube which was wound several times round the outside of the vessel. The progress of the combustion was watched through a window composed of a triple disc of glass, alum, and quartz to prevent loss of heat by radiation. The combustion chamber, with its surrounding coil of tube, was immersed in water contained in a copper vessel forming the calorimeter, which was silver plated on the outside. This was surrounded by a second vessel, silver plated on the inside, and the space between filled with swans' down. The calorimeter was closed by a cover through which a stirrer passed. The silver plating served to prevent loss or gain of heat by radiation. The second vessel was surrounded by a third vessel, the intermediate space being filled with water to prevent variations of the temperature of the air affecting the apparatus.

**64. Calorific Power of Various Substances.** The amounts of heat developed by the combustion in oxygen of one pound of some substances are given in the following table:—

## HEAT OF COMBUSTION IN OXYGEN.

Substance.	Units of heat evolved.	Observer.
Hydrogen	{ 33808 34462	Andrews Favre and Silbermann
Wood charcoal	{ 7900 8080	Andrews Favre and Silbermann
Sulphur	2307	Andrews
Phosphorus	5747	"
Zinc	1301	"
Iron	1576	"
Copper	603	"
Carbonic oxide	{ 2403 2431	Favre and Silbermann Andrews
Marsh gas	13108	"
Ethylic alcohol	{ 6850 7184	" Favre and Silbermann
Welsh coal	about 8241	
Newcastle coal	" 8220	
Derbyshire coal	" 7733	
Wood (dried in air)	" 3547	

From this table it will be seen that the heat developed by the combustion of one pound of coal is capable of converting between 14 and 15 pounds of water at  $100^{\circ}\text{C}$ . into steam at the same temperature.

There are two points of importance which are indicated by this table. The heat developed by the combustion of 1 lb. of pure carbon may be taken as 8000 units. The table shows that the heat developed by 1 lb. of carbonic oxide,  $\text{CO}$ , in burning to carbonic anhydride,  $\text{CO}_2$ , is about 2400 units. Now, only three-sevenths of the pound of carbonic oxide consists of carbon; hence a pound of carbon when already combined with  $1\frac{1}{3}$  lb. of oxygen, will, in combining with an additional  $1\frac{1}{3}$  lb. of oxygen, so as to pass from  $\text{CO}$  to  $\text{CO}_2$ , develop 5600 units of heat. But a pound of carbon in burning directly to  $\text{CO}_2$  develops only 8000 units of heat; hence a pound of carbon in combining with its first  $1\frac{1}{3}$  lb. of oxygen to form  $\text{CO}$  can develop only 2400 units of heat, though it produces 5600 units in combining with the second equivalent of oxygen. Carbonic oxide, therefore, possesses about seven-tenths of the calorific power of the carbon it contains. Hence the great calorific power of blast furnace gases.

The table indicates that the calorific power of marsh gas may be taken as 13100, and that of hydrogen as 34000. Now, 1 lb. of marsh gas contains 12 ozs. of carbon and 4 ozs. of hydrogen. The calorific value of 12 ozs. of carbon is 6000 units of heat, and that of 4 ozs. of hydrogen 8500 units of heat; so that from its composition we might expect the calorific power of marsh gas to be 14500, instead of 13100, as experiment shows it to be. The difference, 1400 heat units, represents the work which has to be done in order to decompose the marsh gas,—that is, to separate the carbon from the hydrogen before either can combine with the oxygen.

65. These results indicate that the ultimate analysis of a fuel is not a safe guide to its calorific power. In estimating the calorific power of coal from its ultimate analysis, it is usual to suppose that the oxygen contained in the coal is in combination with its due portion of hydrogen, forming water, and that the rest of the hydrogen is com-

bined with carbon, as marsh gas. These hypotheses will generally give a result somewhat below the truth.

For example, we may take a sample of Newport steam coal containing in 100 parts :

Carbon	81.47
Hydrogen	4.97
Nitrogen	1.63
Oxygen	5.23
Sulphur	1.10
Ash	5.51

The oxygen, if combined with hydrogen, will have associated with it .67 parts out of the 4.97 parts, thus leaving 4.3 parts of hydrogen to be in combination with the carbon, while the 5.9 parts of water formed by the oxygen and hydrogen have no calorific power. The 4.3 parts of hydrogen, if in the form of marsh gas, will be in combination with 12.9 parts of carbon, forming 17.2 parts of marsh gas, and leaving 68.57 parts of free carbon. Taking 100 lbs. of coal we have :

68.57 lbs. of carbon, giving	548560	heat units,
17.2 lbs. of marsh gas „	225320	„ „
1.1 lb. of sulphur „	2530	„ „

in all, 776410 heat units, or about 7764 heat units per pound of coal. This corresponds to an evaporative power of nearly  $14\frac{1}{2}$  lbs. of water at  $100^{\circ}$  C. converted into steam at the same temperature, but it is somewhat below the calorific power of either the Welsh or Newcastle coal given in the table. Had all the hydrogen in the above sample of coal been free, its calorific power would have been 8232 heat units, and its evaporative power about 15.4.

In a very good steam boiler a pound of good steam coal will produce from 9 to 10 pounds of steam at a temperature of, say,  $150^{\circ}$  C., the temperature of the feed water being about  $50^{\circ}$  C. This corresponds to an efficiency on the part of the boiler of from 67 to 75 per cent. The rest of the heat developed is lost, mainly with the products of combustion passing up the shaft.

66. **Loss of Heat in Chimney Draught.** One very  
G. H. E

important consideration in connection with steam boilers is the amount of air necessary for the combustion of the coal, and the consequent loss of heat through the products of combustion. If forced draught be employed, whether the draught is produced by a steam blast or fan, there is no reason why the products of combustion should not be cooled down almost to the temperature of the water in the boiler, provided that it is possible to introduce into the boiler sufficient heating surface. But when a chimney is relied upon for the production of the draught, it is necessary that the products of combustion should, on entering the chimney, possess a sufficiently high temperature to create the necessary draught. This temperature will be less the higher the chimney, provided that the diameter of the shaft is sufficient to render its own friction of small importance. If a chimney is too short or too small in diameter, a great waste of heat is entailed in producing the necessary draught; and it should be noticed that for this purpose each additional unit of heat employed is less effective than the preceding until the temperature in the chimney is about  $320^{\circ}\text{C}$ ., when the draught is a maximum if the resistance is due only to the chimney. It is therefore of great importance properly to design the chimney of a boiler-house, so that the resistance of the chimney may be small in comparison with that of the fires and the flues or tubes.

67. For example, in the case of the steam coal referred to above, 2.53 lbs. of oxygen would be required for the complete combustion of each pound of coal. This means almost exactly 12 lbs. of air; but as it is impossible to remove the whole of the oxygen from the air in an ordinary furnace, it will be found necessary to supply about 14 lbs. of air per pound of coal to secure satisfactory combustion. If we suppose the air to enter the furnace at a temperature of  $12^{\circ}\text{C}$ ., and to enter the chimney at  $360^{\circ}\text{C}$ ., and if we further suppose the capacity for heat of the products of combustion to be the same as that of the entering air, we have for the number of units of heat employed in heating the products of combustion of each pound of coal, and escaping up the chimney,  $348 \times 14 \times .24$ , that is, 1169.28; or, of the 7764 heat units produced,

about 15 per cent. is lost with the escaping products of combustion.

68. Unless the air supply is very carefully regulated, it will be found that considerably more than 14 lbs. of air will pass through the furnace for every pound of coal consumed. Sometimes the amount of air exceeds 26 lbs. per pound of coal, and if the temperature of the "uptake" is very high, as is usually the case with forced draught, the loss of heat in the escaping gases becomes very serious.

The difference between the pressure due to the heated gases in the chimney, and that of a column of atmospheric air of the same height at the temperature of the surrounding air, is the pressure which drives the air through the furnace, overcomes the friction of the flues and chimney shaft, and communicates to the air the velocity with which it is finally discharged. By employing a shaft of sufficient diameter the velocity of the issuing gases and the friction of the shaft can be made very small. The principal resistance to be overcome then is that of the furnace bars, the fires, and the boiler flues or tubes. As a rule, a pressure equivalent to about  $\frac{1}{6}$  inch of water is an efficient working pressure; but of course this depends very much on the design of the boiler. After reading Chapter V., the student will be able to show that if the height of the chimney above referred to be 90 feet, and the mean temperature of the gases in it be  $350^{\circ}$  C., their density being supposed to be the same as that of air at the same temperature, the effect of the chimney will be equivalent to a water-gauge of  $\frac{1}{2}$  inch.

69. (III) **Mechanical Sources of Heat.** Of the mechanical sources of heat, friction is perhaps the best known. A very common experiment consists in making a button hot by rubbing it against a form, and it is no rare occurrence for a railway axle to become red-hot through the supply of grease becoming exhausted, or otherwise failing in its duty. The bearings of the rollers employed in rolling bar iron, and which are subject to very great pressure, are kept cool by a stream of water continually running over them, and this also serves to

lubricate the surfaces. Most persons have observed the sparks which fly from beneath the wheels of a brake-van when the brake is applied so as to prevent their rotation. If a small brass tube be filled with water and corked, and be then made to rotate rapidly while it is squeezed between two pieces of wood, the heat caused by the friction will soon cause the water to boil, and to violently eject the cork. Count Rumford raised the temperature of 18·77 pounds of water, besides a large quantity of metal, from  $15^{\circ}$  C. to  $100^{\circ}$  C. in  $2\frac{1}{4}$  hours by means of the friction of a blunt borer in a gun-metal cylinder. (See Chapter XI.) If a piece of tin be bent in opposite directions in rapid succession, a crackling noise is heard and a great amount of heat generated.

70. Heat is developed when work is done by compressing a substance, whether the pressure act for a long time or for an exceedingly short time, though in the former case the heat, having a long time in which to escape, does not always make itself manifest. A few blows with a hammer will raise the temperature of a piece of lead very considerably, and a dexterous smith may heat a small piece of iron to redness by hammering it. The splinters which fly from a flint when struck by a piece of hard steel are often at a white heat. The impact of a chilled iron shot on an iron target produces a great amount of heat together with a bright flash of light. If air contained in a cylinder be suddenly compressed by the descent of a piston, it may be readily heated sufficiently to ignite German tinder, or the vapour of bisulphide of carbon, previously introduced into the cylinder. When a drop of bisulphide of carbon has been introduced, the sudden compression of the air is accompanied by a bright flash of light.

71. That heat is developed when work is done against the resistance of a body to torsion is readily shown by the following experiment. A brass tube, four or five inches in length and about  $\frac{3}{8}$ ths of an inch in diameter, is soldered at both ends to cross-pieces, one of which is drilled through so that the bulb of a thermometer may be inserted into the tube. One of the cross-pieces being



fixed in a vice, the other is rapidly turned through a few degrees in opposite directions successively. The heat generated in continually reversing the torsional strain produces in a few minutes a very appreciable rise of temperature in the brass tube, which is rendered apparent on the insertion of the thermometer.

The heating effect of a direct shearing strain is conspicuous in the punching of holes in boiler or other plates, and in the shearing of iron bars. Generally the newly sheared surfaces are far too hot to be touched by the hands.

72. There are a few bodies which contract when heated, and expand on cooling. Such bodies become heated when caused to expand by an external agent, and heat is absorbed by them on contracting. A piece of stretched india-rubber when heated contracts in the direction of its tension. This may be easily shown by suspending a weight by means of an india-rubber tube, a small piece of glass or metal tube being inserted at each end in order that a string may be attached without closing the tube. If a current of steam be blown through, the india-rubber will contract, thus raising the weight, and will expand again as the tube cools. For the success of this experiment it is necessary that the tube should be new and in good condition. Tubes which have been exposed for some months to the air will not show the effect. New red rubber tubing answers very well. If a piece of india-rubber be suddenly stretched and applied to the face of a thermo-pile,\* heat will be at once indicated; while if the rubber be allowed to contract, exerting tension all the time, the face of the thermo-pile in contact with it will be cooled.

Whenever a body expands on being heated, heat will be generated if it is compressed by an external agent; but if the body contract when heated, heat will be generated by forcibly causing it to expand.

73. Whenever work is done or energy lost by a me-

\* See Chapter VII.

chanical system, and there is no form of energy other than heat into which it can be converted, the whole of the work so done or energy so lost has its equivalent in heat generated in the system. This is true whether the bodies we are dealing with are large or molecular, so that we may expect heat to be generated whenever a state of strain in any body is relieved.

74. (IV) **Heat from Electrical Energy.** Whenever an electric current is sent through any resistance, heat is produced. Dr. Joule has shown that the heat produced is proportional to the product of the resistance and the square of the current. If a strong current be sent through a wire of considerable resistance, but whose capacity for heat is very small, the temperature of the wire will be very much raised, and it is easy in this way to fuse a few inches of fine platinum wire by the current from two or three Grove's cells. Electric lights illustrate the same effect. A current may be produced by causing a coil of wire to rotate between the poles of a magnet, and when the ends of the coil are connected together without the introduction of any fine wire, the whole of the heat is generated in the coil itself. When a Grove's or bichromate battery has its poles connected by a short thick copper wire, a great current is produced, and a quantity of heat proportional to the square of the current; but nearly the whole of this heat is produced in the cell itself. When a piece of brass or copper is made to spin between the poles of a powerful magnet, great resistance to the motion is experienced, and heat is generated on account of electric currents induced in the metal. If iron be employed instead of brass or copper, the iron becomes magnetized first in one direction and then in another, and the successive reversal of the magnetization adds to the heating effect (Hysteresis). If a strongly charged Leyden battery be discharged through a fine metallic wire stretched on a card, say a foot of wire .002 inch in diameter, the wire will be dissipated, forming a beautiful streak on the card, with here and there a system of dark lines radiating from a point which looks like a centre of explosion. If the wire be covered with

silk, the latter will remain intact, though the wire is dissipated between the threads.

75. The amount of heat generated in a wire is always equal to the product of the current and the electro-motive force between the ends of the wire; that is, to the work done by the battery, or other source of current, against the resistance of the wire.

The heat generated by a very great current has recently been employed for the welding of iron, steel, and other metals; for the melting of steel, platinum, etc., and for the direct reduction of aluminium from Bauxite. If a piece of stout brass wire be bent into a circle, so as to embrace the core of a Gaulard & Gibbs' transformer, its ends being joined, the wire will be heated to redness by the currents induced in it. Professor Elihu Thomson's welding machine is similar to this apparatus in its action. A series of copper bars are laid together and bent into the form of a horse-shoe, the ends being attached to heavy copper clips suitable for holding the bars to be welded, and provided with screws by which the ends of the bars can be pressed together. Currents of several thousand *ampères* are generated in this horse-shoe, and pass across the junction of the bars at which the weld is to be made, raising the ends of the bars to a welding heat. The currents are generated by means of comparatively small alternating currents in a coil of a great many turns which lies close to the copper horse-shoe, the induction being increased by a considerable mass of soft iron wire wound transversely round the coil and the horse-shoe.

76. When an electric current passes across the junction of dissimilar metals, heat may be produced or absorbed according to the direction of the current. This is known as the Peltier effect. When a current passes along a metal, the temperature of which varies from point to point, heat may be absorbed or produced according to the direction of the current and the character of the metal. This is known as the Thomson effect. (See Chapter VII.)

77. (V) If the rapidly alternating currents from an alternating current dynamo be sent round the coil of an

electro-magnet, the iron core becomes rapidly heated, but this is not the case if a steady current of the same strength as the alternating currents be caused to flow continuously through the coils. The cause of the heating is partly the rapid variation of the magnetism of the iron, but is mainly due to currents induced in the iron core.

In all well designed transformers and in the armatures of dynamos, the iron cores are built up of thin sheets of iron or of iron wire so as to prevent induced currents circulating in them. The heating of the core is then due almost entirely to the rapid magnetization and demagnetization of the cores, an effect known as hysteresis. It is not unusual for three per cent. of the total output of a transformer to be lost in this way, and sometimes more than one per cent. of the power given to a dynamo is lost from the same cause. The effect is less the softer the iron. It is very great with hard steel.

78. (VI) **Latent Heat.** Whenever heat is consumed during any transformation, as when a body passes from the solid to the liquid state, heat will be produced when the transformation takes place in the opposite direction. Thus heat is given out by water in freezing, and if 100 grains of steam at  $100^{\circ}\text{C}$ . be passed into 536 grains of water at  $0^{\circ}\text{C}$ ., we obtain 636 grains of water at  $100^{\circ}$ ; so that the steam raises the temperature of the water by the heat it emits in condensation without changing its own temperature.

79. Certain saline solutions, such as a solution of sodium sulphate, if saturated at a high temperature and allowed to cool in a closed vessel, or in a flask whose neck is plugged with cotton wool, can be reduced to the ordinary temperature, without crystallization taking place in the same way as it would if the solution were exposed. On introducing into the solution when cold certain substances, some of which are always to be found in the air, but the nature of which has not yet been clearly made out, a mass of crystals is produced extending throughout the solution, and the temperature rises very considerably. In fact, the "super-saturated solution," as it is called, is

in a state of strain, and this strain is suddenly relieved when crystallization takes place, the work done by the crystallizing forces being represented by the heat generated. Water which has been freed from air by long boiling may be cooled considerably below the freezing-point without congelation ; but on being disturbed a network of ice crystals is formed throughout the water and heat is produced (equivalent to the latent heat of the water which freezes), while the temperature is raised to  $0^{\circ}$  C.

80. Iodide of mercury, if heated, assumes a new crystalline form, and changes its colour from red to yellow. It retains this form when cold if it be not subjected to mechanical violence, but on crushing the crystals they change to the red variety with evolution of heat, and the change spreads gradually throughout the whole mass. Boiling sulphur poured into cold water assumes a plastic form, and can be stretched like india-rubber. This material gradually changes to an opaque crystalline mass, and heat is evolved during the transformation. Sometimes the formation of each crystal in a hot saturated solution of salts is accompanied by a flash of light.

Yellow phosphorus, when heated nearly to its boiling point, and maintained at this temperature for several hours, assumes the red or amorphous condition, and much heat is absorbed during the operation. If this red phosphorus be heated too much, it returns to the yellow variety with explosive violence, and the heat absorbed during the former transformation is restored.

81. **Solar Radiation.** If we trace back any supply of heat as far as possible, we find that, if we neglect such unimportant sources as meteors, we always arrive either at the internal heat of the earth, the earth's motion of rotation converted into heat by the intervention of the tides and machinery driven by them, or the radiation from the great centre of our system, the sun. Take, for example, the heat generated in a wire through which an electric current due to a zinc-carbon battery is passing. The heat is derived from the energy of the current, which is itself derived from the combination of the zinc with

the exciting liquid. Now this energy is due to the attraction of the zinc for certain components of the liquid, and for the energy of this attraction the zinc is indebted to the heat which separated it in the reducing furnace from its combination with oxygen, etc., in the ore. This heat is due to the combustion of the coal, and therefore to the attraction which the coal possesses for oxygen; and for this attraction the coal is indebted to the energy of the solar radiation, which, acting through the machinery of a vegetable organism, separated the carbon, etc., from combination with oxygen at the time when the trees grew from which our coal has been produced.

82. It is very easy to measure the total energy of the solar radiation received upon, say, one square foot of the earth's surface in ten minutes, and from this it would be easy, if the sun's distance were accurately known, to calculate the whole amount of energy radiated by the sun in any given time if there were no absorption by the earth's atmosphere. It is, however, very difficult to estimate the amount of this absorption. Comparisons between simultaneous measurements made at the top and at the foot of lofty mountains, and between measurements made at the same place when the sun is high in the heavens and when his altitude is much less, have served to indicate on the one hand the absorption of the lower layers of the earth's atmosphere, and on the other the effect of increasing the thickness of the layer of air, but it is difficult to deduce from these measurements precisely what the effect would be if the atmosphere were removed altogether, because, as will be pointed out again in the chapter on radiation, it generally happens that when radiant energy has passed through a certain thickness of a medium, the rays which that medium can most readily absorb become filtered out, and the remaining radiation passes through successive layers of the same medium with much less absorption. Recent experiments conducted on Mount Whitney, in the Sierra Nevada, where the air is exceedingly dry and transparent, indicated that the absorption by the air was much greater than had previously been believed. It thus appears that the solar

radiation is equivalent to the energy of combustion of more than a ton of coal per hour, on every square foot of the surface of the sun's photosphere. With a forced draught it is possible to burn about 130 lbs. of coal per hour on every square foot of grate in a marine boiler. With ordinary draught the coal consumed is not more than one-fifth of this amount, and does not generally exceed 20 lbs. per square foot of grate surface; so that if all the heat generated in the furnace of an ordinary steam boiler were radiated from a surface equal in area to the grate, the radiation would scarcely exceed, if it did at all exceed, one per cent. of the radiation from an equal area of the sun's photosphere. The heat radiated by one square foot of the sun's surface is distributed on the average over rather more than four acres of the earth's surface,—the equatorial regions, however, getting nearly double the average share. Thus, taking the average over the earth's surface, it would require the consumption of more than 2000 tons of coal per acre per annum, to supply energy equivalent to the solar radiation, though it must be admitted that a considerable part of the solar energy is absorbed by the air, and only benefits the earth indirectly.

83. If the sun were a solid mass of coal, and oxygen for its combustion were supplied from some other source, the heat produced would supply the solar radiation, as at present taking place, for only about 5000 years. Two other possible sources of the sun's energy have been suggested,—the falling of meteoric bodies or of small planets into the sun, and the gradual contraction of the sun itself under its own gravitation. The determination of the mechanical equivalent of heat enables us to calculate the heat generated by any mass falling into the sun from a given distance, or the heat equivalent of the work done by the gravitating forces if the sun's mass contract by any specified amount. How far the first-mentioned cause is efficient may be judged from the fact that if all the known planets were to fall from their orbits into the sun, the heat generated would supply solar radiation for about 40,000 years. In the possible con-

traction of the sun itself we have a much greater reserve of energy. At present the sun's mean density is only about one-quarter that of the earth, reckoning the surface of the photosphere as the sun's external surface. If he were to contract to one-half of his present diameter, and therefore one-eighth of his present volume, his mean density would then be only double that of the earth, while the work done by gravitation during the contraction would supply energy for the solar radiation for more than four millions of years.



## CHAPTER IV.

### EFFECTS OF HEAT UPON MATTER.

84. **Solids and Fluids.** BODIES may be divided into two classes, *solids* and *fluids*.

DEF. *A solid is a body which requires a finite stress to produce a finite change in its form with or without change of volume.*

DEF. *A perfect fluid is a body whose form can be changed to any extent, provided its volume remain constant, by the application of a stress, however small, if we allow it sufficient time.*

One solid is said to be more *rigid* than another, when, other things being the same, it requires a greater stress to produce in it the same change of form. A solid would be *perfectly rigid* if no finite stress, however great, could produce any change whatever in its form or volume. We know of no body perfectly rigid or perfectly fluid.

As illustrations, we may refer to a piece of hard steel, which requires a very great force to produce a permanent impression upon it; while its own weight is sufficient to make any quantity of water acquire the form of the vessel into which it is poured.

A body is said to be *elastic* when, if strained, it tends to return to its original form or volume after the constraint has been removed. Fluids are elastic for changes of volume only, and have no tendency to assume any particular shape. (The spheroidal form assumed by drops of liquid is due to the combined action of gravity and a tension of the *surface* of the liquid, known as *surface*

*tension*, to which reference has already been made in describing maximum and minimum thermometers. Capillary action is due to the same cause.)

There are many substances which occupy a kind of intermediate position between solids and liquids,—as, for example, many vegetable gums, etc.,—and are called *viscous*.

85. **Liquids and Gases.** Fluids are divided into two classes, viz., *liquids* and *gases*.

DEF. *A liquid is a fluid, a finite portion of which cannot be made to occupy more than a certain definite volume, however the pressure to which it is exposed may be diminished.*

DEF. *A gas is a fluid, any finite portion of which may be made to occupy any space, however great, by sufficiently diminishing the pressure to which it is exposed.*

For example, if a cup of olive oil, or strong sulphuric acid, be placed under the receiver of an air-pump, the liquid will not sensibly change its volume, however perfect a vacuum may be formed around it; while if a cubic inch of air at ordinary pressure be allowed to enter an exhausted receiver of *any* capacity, the air will expand and completely fill the receiver, exerting an uniform pressure all over its surface, as may be shown by the same fall being produced in columns of mercury in manometer tubes communicating with the receiver at different points. This power of indefinite expansion is the essential characteristic of a gas.

86. According to the molecular theory of gases now generally accepted, we may perhaps say proved, a gas consists of perfectly free particles, or molecules, moving in all directions and with different velocities, but such that the energy due to the motion of the particles in any sensible mass of the gas depends only on the temperature, increasing with it. The motion of each particle is unaffected by any other particle except when very close indeed to it, when the particles rebound from each other like perfectly elastic balls. From this view of the constitution of a gas it follows that any finite quantity of a gas will expand

into any portion of space to which it has free access. The pressure exerted by a gas on a surface in contact with it is due to the repeated impact of its particles, in the same way as the pressure exerted by a jet of water from a fire-engine is due to the impact of the several particles of water in the jet. The pressure is of course increased when the velocity is increased, and in the case of the fire-engine jet is proportional to the square of the velocity if the sectional area of the jet be given. So, in the case of gases, the pressure increases with the velocity of the particles, being proportional to the mean square of the velocity; it therefore increases with the temperature, and, as we shall learn in the next chapter, is proportional to the temperature as recorded by the air thermometer, whose zero is  $-273^{\circ}\text{C}$ .

**87. Expansion by Heat.** We have seen that the effect of the entrance of heat into bodies is in general to increase their temperature, and this is *usually* accompanied by increase of volume, if the pressure to which they are subject be not increased so as to prevent it. Water between the temperatures of  $0^{\circ}\text{C}$ . and  $4^{\circ}\text{C}$ ., iodide of silver, baked clay, and one or two alloys, are, however, notable exceptions to this rule, since they contract on increase of temperature.

Among solids the metals as a general rule expand most for a given increase of temperature, but ebonite expands much more than any metal.

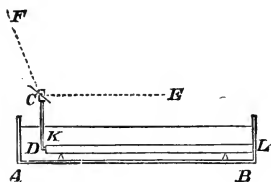
**DEF.** *The ratio of the increase in length of a bar of any substance, due to an increase in its temperature of  $1^{\circ}\text{C}$ ., to the original length of the bar at  $0^{\circ}\text{C}$ ., is called the coefficient of linear expansion of the substance.*

Thus, if the length of a bar of copper is one foot at  $0^{\circ}\text{C}$ . and 1.0017 feet at  $100^{\circ}\text{C}$ ., its mean coefficient of expansion between  $0^{\circ}\text{C}$ . and  $100^{\circ}\text{C}$ . is .000017; for if the expansion were uniform between  $0^{\circ}\text{C}$ . and  $100^{\circ}\text{C}$ ., its length at  $1^{\circ}\text{C}$ . would be 1.000017 feet, and the ratio of the expansion to the original length would then be .000017.

**88.** The linear expansion of bars may be conveniently measured by means of the apparatus shown in Fig. 10.

$AB$  is a bath of oil or water which can be heated to any required temperature, and the temperature observed by means of a thermometer.  $CD$  is a small bar which can turn about a centre at  $C$ , and carries a blunt knife-edge at  $D$ , resting against the end  $K$  of the bar  $KL$ , whose expansion is to be observed. The arm  $CD$  carries a small mirror near  $C$ , and the distance between the centre of motion  $C$  and the knife-edge at  $D$  is accurately known. The end  $L$  of the bar  $KL$  rests against a fixed obstacle, as, for instance, the end of the vessel, and the centre  $C$  and end of the vessel  $B$  should be so supported that the distance between  $C$  and  $L$  is invariable. The adjustment should be so made that during the experiment the mean position of the line  $CD$  is as nearly as possible vertical. A ray of light is allowed to fall on the mirror in the direction  $EC$ , which remains constant, and is reflected along  $CF$ , forming a spot of light on a scale conveniently situated. When the bar  $KL$  is heated, it expands, and the end  $K$  pushes  $D$  before it, since  $L$  is fixed. This causes the arm  $CD$ , and with it the mirror, to turn about  $C$ , the reflected ray  $CF$  consequently being deflected through twice the angle through which  $CD$  turns. This angle is indicated by the deflection of the spot of light on the scale, and hence the angle through which  $CD$  turns, and therefore the distance through which  $D$  moves, or the linear expansion of  $LK$ , can be at once determined.

FIG. 10.



This method is similar to that of Lavoisier, who employed a telescope attached to the arm  $CD$  instead of the mirror, and through it observed a fixed scale. The mirror has the advantage of being much lighter than the telescope, and therefore more easily mounted, while it deflects the ray through twice the angle through which it turns.

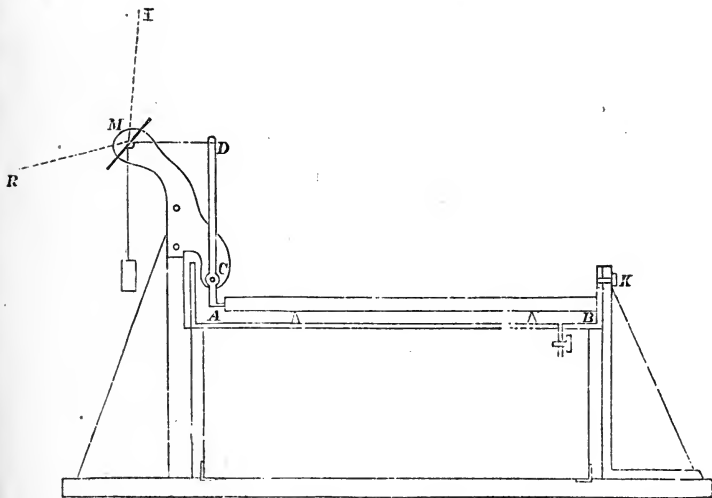
89. In order to make accurate measurements with the mirror, a horizontal wire should be stretched in front of the source of light, and a lens placed between the wire and the mirror, so that a distinct image of the wire is thrown upon the graduated scale. The scale should be circular,

and have the mirror at its centre. The motion of the image of the wire will then at once indicate the angle through which the mirror is turned.

By causing the end *B* of the bar to press against a micrometer screw inserted in the end of the trough, we can find how many turns of the screw produce the same deflection as the expansion of the bar when heated, and thus determine approximately the actual expansion.

90. To show the expansion of metals for purposes of demonstration, the apparatus shown in Fig. 11 is very

FIG. 11.



convenient, though it cannot be used for exact measurements on account of the extensibility of the string by which the mirror is moved. The mirror, instead of being directly connected with the lever at *C*, is carried on a separate shaft at *M*. The pivot *C* is placed very near to the lower end *A* of the lever, and to the upper end of the lever a string is attached which passes round a small spindle carrying the mirror and supports a weight at the end, or is attached to a steel spring which keeps it stretched. In this way the angle through which the

mirror turns is very much increased. It is important that the pivot  $C$  should be rigidly fixed with respect to the end  $B$  of the rod, and this is ensured by the strong brackets shown in the figure. The bath is supported upon pieces of wood fastened to the stand by hinges for the convenience of removing it. If the diameter of the mirror spindle at  $M$  be  $\frac{3}{16}$  in., the length of  $CD$  3 inches, that of  $AC$   $\frac{1}{2}$  in., and of  $AB$  12 inches, then, if the bar be zinc, a change of temperature of about  $64^\circ$  Centigrade will turn the reflected ray through  $180^\circ$ , but if the bar be iron, the same change of temperature will turn the ray through only  $72^\circ$ ; showing that the coefficients of expansion of zinc and iron are nearly as 5 : 2.

91. The mean coefficients of linear expansion of some substances between  $0^\circ$  C. and  $100^\circ$  C. are given in the following table.

TABLE OF COEFFICIENTS OF LINEAR EXPANSION.

Glass tube	about	·000008
Crown glass	"	·000009
Platinum	"	·000009
Cast iron	"	·000011
Soft steel	"	·000011
Steel tempered yellow	"	·000013
Wrought iron	"	·000012
Gold	"	·000016
Copper	"	·000017
Brass	"	·000019
Silver	"	·000021
Tin	"	{ ·000022 to
		{ ·000028
Lead	"	·000028
Zinc	"	·000030

The coefficients of expansion of metals vary very much with their physical state, so that the above table must be taken as only approximately true.

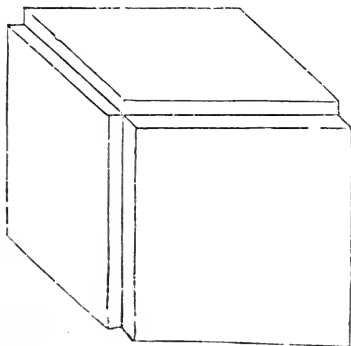
92. DEF. *The coefficient of cubic expansion of a substance at any temperature is the ratio of the increment of any volume of the substance produced by an increase in its temperature of  $1^\circ$  C. to the original volume at  $0^\circ$  C.*

Most substances, except crystals, expand equally in all directions when heated. Imagine a cubic block, the length of whose edge is 1 foot : its volume will then be 1 cubic foot. Now suppose that on raising its temperature  $1^{\circ}\text{C.}$ , the length of its edge becomes  $1+a$  feet, so that  $a$  is its coefficient of linear expansion. Then its volume is  $1+3a+3a^2+a^3$  cubic feet, and the increment of its volume is  $3a+3a^2+a^3$  cubic feet. The ratio of this to the original volume, viz., one cubic foot, is  $3a+3a^2+a^3$ , which expression is therefore the coefficient of cubic expansion. Now  $a$  is, in general, very small; hence  $a^2$  and (*a fortiori*)  $a^3$  may be neglected in comparison with  $a$ . We have then for the coefficient of cubic expansion  $3a$ , or *the coefficient of cubic expansion is three times the coefficient of linear expansion* for the same substance.

In the case of copper  $a$  is about  $\cdot000017$ . Hence  $3a$  is about  $\cdot000051$ ,  $3a^2$  about  $\cdot000000000867$  and  $a^3$  about  $\cdot000000000000004913$ . It is obvious that both  $3a^2$  and  $a^3$  may be neglected without producing any sensible error in the result, each being far less than the limits of experimental error in the determination of  $a$ .

93. The effect of neglecting the terms involving  $a^2$  and  $a^3$  may be illustrated by taking a cube of 10 centimetres side, three plates each 10 centimetres square and one centimetre thick, three strips each 10 centimetres long and one centimetre square and a cubic centimetre.

FIG. 12.



Placed together these will build up a cube of 11 centimetres edge. If we neglect the three strips and the cubic centimetre, our enlarged cube is incomplete at the edges (Fig. 12), and this is equivalent to neglecting  $3a^2$  and  $a^3$  in the above expression.

Or we may employ the following illustration. Take a cube of wood or other material of one foot edge, and let this represent the unit of volume. Suppose  $a$  to be  $\cdot 01$ . Then a piece of pasteboard of one foot square and  $\cdot 01$  ft. thick will contain  $a$  units of volume, while its thickness will be  $a$  units of length. Take three such plates, the sum of whose volumes is  $3a$ , and apply them to the three faces of the cube which meet in a point. Now take three rectangular strips of pasteboard a foot long and  $\cdot 01$  ft. square. The volume of each of these is  $a^2$ , and the volume of the three together is  $3a^2$ . If these strips be laid in the grooves formed by the edges of the plates, there will only be required a cube of  $\cdot 01$  ft. side and volume  $a^3$  in order to complete a cube of  $1\cdot 01$ , or  $1 + a$ , feet edge. The whole increment in volume of the one foot cube is the volume of the three plates together with that of the three strips and the small cube, or  $3a + 3a^2 + a^3$ . If we take the coefficient of cubic expansion to be three times that of linear expansion,—that is, take into account the plates only,—we neglect the strips and the little cube; but even when the linear expansion is so great as  $\cdot 01$  of the original length, the error so introduced is very little more than one per cent.

94. A *homogeneous* substance is one whose structure is the same throughout, so that spheres of the same size cut from different portions of the substance are alike in all respects.

An *isotropic* body is one whose substance has the same properties in all directions, so that if a sphere were cut from the body it would not be possible to determine by any test whatever the direction which a particular diameter of the sphere occupied in the body. Substances which are not isotropic are called *æolotropic*.

An isotropic substance expands equally in all directions, and its coefficient of cubic expansion is then, as we have



seen, three times its coefficient of linear expansion. But very few solid substances are really isotropic. Some cast metals are very nearly so, and carefully annealed glass is perhaps as close an approximation to an isotropic substance as any solid we are likely to meet with, but it must be very carefully annealed. In the case of metals, almost any mechanical operation, such as rolling, hammering, wire-drawing, etc., renders them very far from isotropic, though in the case of some metals careful annealing may go far to restore their original condition. Boiler plates, for example, possess very different properties along the plate (in the direction of rolling), across the plate, and through the plate. In particular, the tensile strength of an iron boiler plate is much greater when the tension is in the direction of the fibre than when it is across the fibre, and plates which have a tensile strength of 47,000 lbs. per square inch in the direction of the fibre, have a strength of only 40,000 lbs. per square inch across the fibre. In general, an æolotropic substance, when heated, expands differently in different directions, and has an infinite number of coefficients of linear expansion according to the direction in which the length is measured. A sphere cut from such a substance will generally expand, when heated, into an ellipsoid. The axes of an ellipsoid are its greatest diameter, its least diameter, and a third diameter which is at right angles to each of these. The volume of an ellipsoid is to the volume of a sphere as the product of its three axes is to the cube of the sphere's diameter, and its volume is therefore expressed by  $\frac{4}{3} \pi abc$  where  $a, b, c$  represent the semi-axes. If a rectangular parallelepiped is cut with its edges parallel to the axes of the ellipsoid of expansion, the parallelepiped will, when heated, remain rectangular; but if its edges are not in these directions, it will on expansion become oblique.

95. Suppose a unit cube to be cut with its edges parallel to the axes of the ellipsoid, and let  $\alpha, \beta, \gamma$  be the coefficients of expansion in the direction of the edges. The cube then expands into a rectangular parallelepiped whose edges are respectively  $1 + \alpha t$ ,  $1 + \beta t$ ,  $1 + \gamma t$ , and its volume will therefore be  $(1 + \alpha t)(1 + \beta t)(1 + \gamma t)$ . But if we neglect the products two and three together of  $\alpha t$ ,

$\beta t$ ,  $\gamma t$ , this becomes  $1 + (a + \beta + \gamma)t$ , and the coefficient of volumetric expansion is  $a + \beta + \gamma$ , or the sum of the principal coefficients of linear expansion.

In the case of stretched india-rubber, one of the coefficients,  $a$  say, is negative, and the other two,  $\beta$  and  $\gamma$ , are positive; but  $a$  is less than  $\beta + \gamma$ , so that  $a + \beta + \gamma$  is a positive quantity, and the volume of the india-rubber is increased by heating it. In this case there will be a certain cone such that the material never expands nor contracts in directions parallel to its generating lines.

**96. Expansion of Liquids.** Before explaining the methods of measuring directly the cubic expansion of solids, we must consider the cubic expansion of liquids; for as the volumes of irregular solids can only be determined from the amount of liquid which they displace, so we should expect that any changes in their volumes must be measured in the same way. The usefulness of liquids for this purpose depends on the property in virtue of which any quantity of a liquid can be made to assume the form of a regular prism or cylinder by pouring it into a vessel of that shape, and its volume can then be computed by measuring its *linear* dimensions.

If a liquid be heated in a glass vessel, both the glass and the liquid expand. The *apparent* expansion of the liquid in the vessel is due to the difference between the expansion of the liquid and that of the interior of the vessel. There are two methods in general use for the measurement of this expansion. The first consists in filling with the liquid the bulb and part of the stem of a thermometer tube, the capacity of the bulb and different parts of the length of the stem at some particular temperature being accurately known. By observing the height of the liquid in the stem at the first temperature, the volume of liquid at that temperature is known. The liquid is then heated, and the height again observed, whence the *apparent* increment of volume can be found.

**97.** In the second method a specific gravity bottle is usually employed. This consists of a glass bottle, into which a stopper, having a small hole drilled through it, is accurately ground. The bottle having been weighed is

completely filled with the liquid whose expansion is to be measured, and whose temperature is known. The stopper is then inserted, when the excess of liquid escapes through the perforation, and is carefully wiped away. The bottle and contents are then weighed, and deducting the weight of the bottle from the result of this weighing, we have the weight of the contents. The temperature of the bottle and liquid is then raised, and as the liquid expands it escapes through the stopper, and is wiped off. On again weighing the bottle we determine the amount of liquid which has escaped, and the ratio of this to the amount remaining is the ratio of the whole increment in volume between the two temperatures to the original volume.

98. A very simple and most useful form of the specific gravity bottle is the weight thermometer. This consists of a glass tube (Fig. 13) drawn out to a fine neck, which is bent over so that it can be made conveniently to dip into a small vessel of the liquid with which it is to be filled. The tube is first weighed when empty, and then by alternately heating it so as to expel the air, and cool-

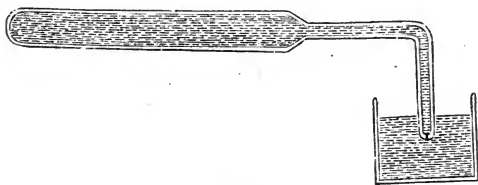


FIG. 13.

ing it with its orifice under the surface of the liquid, the tube is completely filled. The tube and its contents (care being taken that it is completely full) are next weighed at ordinary temperatures, then heated through a convenient range of temperature, the liquid which is expelled being allowed to escape, and finally again weighed. We have then all the data for determining the apparent expansion of the liquid.

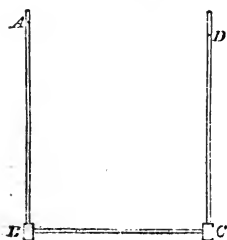
Suppose, for example, that the weight of the empty tube was 120 grains, and that when filled with mercury at  $0^{\circ}$  C. it weighed 720 grains, but that after it had been

heated to  $100^{\circ}$  C., its weight was 710.8 grains. From these data we see that at  $0^{\circ}$  the tube is filled by 600 grains of mercury, while at  $100^{\circ}$  it holds only 590.8 grains, or, in other words, 590.8 grains of mercury at  $100^{\circ}$  occupy the same volume of glass as 600 grains at  $0^{\circ}$ . Hence one grain of mercury at  $100^{\circ}$  occupies (relative to glass)  $\frac{600}{590.8}$  times its volume at  $0^{\circ}$ , or 1.0157... times its volume at  $0^{\circ}$ . The apparent expansion for an increase of temperature at  $100^{\circ}$  C. is therefore .0157 of the original volume, and the mean coefficient of apparent expansion between  $0^{\circ}$  C. and  $100^{\circ}$  C. is .000157.

99. **Absolute Expansion of Liquids.** The apparent expansion of a liquid contained in any envelope is equal to its absolute expansion diminished by the increase in the capacity of the envelope, and therefore the coefficient of apparent expansion is equal to the coefficient of absolute expansion diminished by the coefficient of expansion of the envelope.

Hence if we can determine the absolute expansion of *some particular liquid*, as well as its apparent expansion in a glass vessel due to the same change of temperature, the difference between the two will give us the expansion of the vessel; and then, knowing the apparent expansions of other liquids in the same vessel, we shall only have to add to these quantities the expansion of the vessel, which we have determined once for all, in order to obtain the absolute expansion of all these liquids. The determination of the absolute expansion of some particular liquid is therefore of very great importance, and mercury is the liquid which has been employed for the purpose.

FIG. 14.



100. The method by which this has been effected may be briefly described as follows:

Two tubes  $AB$  and  $CD$  are placed as nearly vertical as possible, the ends  $B$  and  $C$  being connected by a fine tube which is at right angles to the vertical tubes, and therefore in a horizontal position. The tubes are then nearly filled with mercury, and  $AB$  is

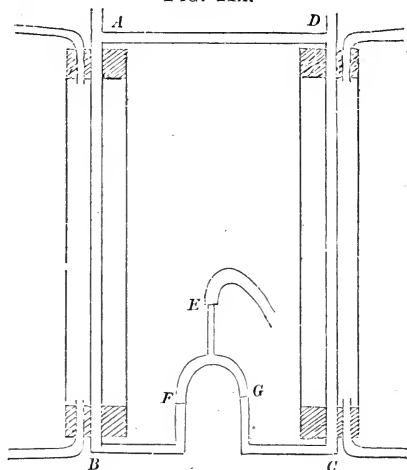
surrounded with a cylinder containing oil, not shown in the figure, which can be heated to the desired temperature. The tube *DC* is surrounded by a cylinder containing melting ice. The heights of the cylinders are so adjusted that the mercury in each tube is just visible above the top of the cylinder surrounding it. When the mercury in each tube has acquired the temperature of the surrounding bath, the heights of the columns *AB* and *CD* are carefully measured. Then, since the mercury in the horizontal tube *BC* is in equilibrium, the pressure at *B* due to the column *AB* is equal to that at *C* due to the column *CD*. Hence the densities of the mercury in these two columns are inversely as their heights; and therefore the volume of a given quantity of mercury at the temperature of either column will be proportional to the height of the column. The ratio of the difference of the heights of the two columns to that of the shorter will therefore represent the total absolute expansion of an unit of volume of mercury between the temperatures of the ice, i.e.  $0^{\circ}\text{C}$ ., and of the oil. The coefficient of expansion for mercury is thus found to be about  $\cdot 00018$ . Its coefficient of expansion at  $0^{\circ}\text{C}$ . is about  $\cdot 000179$ , and increases with the temperature.

A knowledge of this coefficient is necessary to enable us to reduce readings of the barometer to what would be the corresponding readings if the mercury in the instrument were at  $0^{\circ}\text{C}$ .

101. In the apparatus above described the tube *AB* is at a considerable distance from *CD*, and the influence of the bath of hot oil on the refractive index of the air immediately above it renders it very difficult to read the difference of level between *A* and *D* by the telescope of a kathetometer in the usual manner. Regnault surmounted this difficulty by bringing the two surfaces to be compared much closer together and placing them in a position where the air was unaffected by the heat of the oil bath. Figure 14A illustrates a lecture-room model of Regnault's apparatus, in which all the tubes are glass and a steam jacket takes the place of the oil bath.

The tubes *AB*, *DC* are connected by a horizontal tube

FIG. 14A.



at *A* and *D*, and this ensures the pressure in the two tubes being precisely the same at the points where the horizontal tube joins them. The lower ends *B* and *C* are connected to a syphon tube from the highest point of which an india-rubber tube proceeds to a small vessel or reservoir in which air can be compressed by a pump. The tubes *AB* and *DC* are surrounded by much larger tubes closed at the top and bottom by perforated corks, and through the tube surrounding *AB* steam is passed, while cold water flows through the tube surrounding *DC*. The small tubes are filled with mercury and sufficient pressure is applied to the air in the reservoir to cause the surface of the mercury to stand about half-way up each leg of the syphon. If the temperature of the mercury is the same throughout, since it has free passage along the horizontal tube *AD* it will stand at the same level in each arm of the syphon. But when the mercury in *AB* attains the temperature of the steam, and that in *DC* the temperature of the cold water, since the pressures at *A* and *D* remain equal while the column of mercury in *AB* is much lighter than that in *DC*, and the pressure of the compressed air in both legs of the syphon is the same, it is clear that the surface *F* will be lower than the

surface  $G$  by an amount corresponding to the difference of pressure of the columns of mercury in  $AB$  and  $DC$ . Hence by observing the difference of level of the surfaces at  $F$  and  $G$ , the absolute expansion of the mercury can be determined. If the tubes  $AB$  and  $CD$  are about 30 inches in height, and the difference of temperature is  $90^{\circ}$  C., the difference of level between  $F$  and  $G$  will be very nearly half an inch.

**102. Areometric Method.** Having thus obtained sufficient data for determining the absolute expansion of different liquids, we may determine the cubic expansion of a solid by weighing it in a liquid at different temperatures, the coefficient of absolute expansion of the liquid being known; and this process, called the areometric method, is not unfrequently adopted.

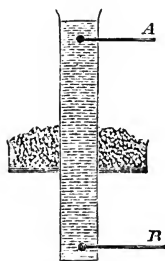
In the same way we may determine the coefficient of expansion of a liquid, if we know the coefficient of cubic expansion of a solid, by weighing a solid in the liquid at different temperatures. Matthiessen adopted this method for determining the coefficient of expansion of water at different temperatures, and some of his results are given in Art. 106. He first carefully measured the coefficient of linear expansion of a glass rod, from which he deduced (Art. 92) the coefficient of cubic expansion. He then cut pieces from this rod, and weighed them in water at different temperatures, and hence deduced the expansion of the water.

**103.** Another method of determining the coefficient of cubic expansion of a solid, when the coefficient of absolute expansion of mercury is known, is by means of the weight thermometer. The apparent expansion of mercury in the glass tube must first be determined, and from this the cubic expansion of the glass is deduced. A bar of the substance is then introduced into the tube before the neck is drawn out, and is supported by means of two pieces of wire twisted round it, and projecting in such a manner as to touch the glass tube only in a few points, otherwise it is impossible to completely fill the space between the bar and the glass with mercury. The volume of the bar is previously determined by weighing

it in air and in water. The neck of the thermometer having been drawn out, it is weighed before and after it is filled with mercury. After heating the thermometer, it is again weighed, and the difference between the amount of mercury expelled, and that which would have been expelled if the thermometer had been filled with mercury, indicates the difference between the expansion of the bar and that of the mercury it displaces. Hence, since the coefficient of expansion of mercury is known, that of the bar can be at once determined.

**104. Expansion of Water.** All bodies, with very few exceptions, expand as their temperature rises, provided they retain the same state of aggregation, *i.e.* remain solid, liquid, or gaseous throughout the change. Exceptions to this law are however met with in baked clay, stretched caoutchouc (which when heated contracts in the direction of its tension), iodide of silver, some crystals (which contract in certain directions while they expand in others) and water between  $0^{\circ}\text{C.}$  and  $4^{\circ}\text{C.}$ , of which the last mentioned is by far the most important. If a quantity of water at  $0^{\circ}\text{C.}$  be heated, it contracts in volume as its temperature rises till it reaches a temperature of nearly  $4^{\circ}\text{C.}$ , when its density is a maximum; if heated beyond this temperature it expands, and at a

FIG. 15.



temperature between  $7^{\circ}\text{C.}$  and  $8^{\circ}\text{C.}$ , its volume is equal to that at  $0^{\circ}\text{C.}$  It then goes on expanding, its coefficient of expansion per degree of the mercurial thermometer increasing continually as the temperature rises. This peculiarity of water may be exhibited by filling a tall cylindrical jar with water, say at  $12^{\circ}\text{C.}$ , small thermometers *A* and *B* being inserted into the liquid. The middle of the jar is surrounded by a freezing mixture.

As the water in the neighbourhood of the freezing mixture is cooled, its density increases, and it descends, causing the thermometer *B* to indicate a rapid fall of temperature, while *A* is very slightly affected. This goes on till the thermometer *B* reaches a temperature of



about  $4^{\circ}\text{C}.$ , when it will cease to fall; but the thermometer *A* will now indicate a fall in temperature, and will continue to do so down to  $0^{\circ}\text{C}.$ , when ice will begin to be formed. This is known as Hope's experiment.

105. We may remark here that water on freezing expands by nearly nine per cent. of its volume, so that the ice formed floats on the surface of the water. This behaviour of water is of immense importance in the economy of nature; for when a quantity of water, as for instance, a lake, cools at the surface, the cold water descends, warmer water from below taking its place; and this goes on till the whole lake has attained a temperature of  $4^{\circ}\text{C}.$ , after which the cold water remains at the surface until a layer of ice is formed there, which prevents the rapid cooling of the remainder of the water on account of the extreme slowness with which it transmits heat. We consequently find that in deep lakes the water below a certain depth is at  $4^{\circ}\text{C}.$ , however cold that at the surface may be. Were water to continue to contract till it froze, we should have the whole of the water in lakes reduced to  $0^{\circ}\text{C}.$  during a continuance of cold weather; and if it also contracted on freezing, we should get an immense quantity of ice accumulated at the bottom of lakes, till the whole lake became frozen; for during the summer the water, warmed by the sun's rays, would float at the top, and the ice below would only be affected by the small quantity of heat transmitted through this water; and thus the ice would increase from year to year, till the whole lake became frozen in the winter, and melted in the summer only to a very small depth.

106. The following table shows the volume of the same quantity of water at different temperatures, according to the experiments of Matthiessen.

$4^{\circ}\text{C}.$	1.000000	$50^{\circ}\text{C}.$	1.011890
$10^{\circ}$	1.000271	$60^{\circ}$	1.016715
$15^{\circ}$	1.000892	$70^{\circ}$	1.022371
$20^{\circ}$	1.001814	$80^{\circ}$	1.028707
$30^{\circ}$	1.004187	$90^{\circ}$	1.035524
$40^{\circ}$	1.007654	$100^{\circ}$	1.043114

107. That water expands, on cooling from  $4^{\circ}\text{C.}$  to  $0^{\circ}\text{C.}$ , may be readily shown in the following manner. Construct a weight thermometer with a long capillary neck. In the first place determine the coefficient of apparent expansion of mercury in the thermometer. Suppose it to be  $\cdot000155$ , while the coefficient of absolute expansion of mercury is known to be  $\cdot000179$ . Then the coefficient of cubic expansion of the glass is  $\cdot000024$ , or about  $\frac{2}{15}$  that of mercury. Hence if the thermometer be filled  $\frac{2}{15}$  full of mercury, the mercury will expand as much as the glass, and the volume of the interior of the thermometer above the mercury will be absolutely constant whatever the temperature. If the thermometer be then nearly filled with water, so that the free surface is in the capillary neck when at  $4^{\circ}\text{C.}$ , the ascent of the surface manifest on lowering the temperature must be entirely due to expansion of the water.

108. **Useful Applications of Expansion and Contraction.** The expansion of metals when heated, and their subsequent contraction on cooling, have been turned to practical account in several of the arts. Thus boiler-plates are rivetted with red-hot rivets, which on cooling contract, and draw the plates together so tightly as to form a joint impervious to high-pressure steam. Again, tyres for ordinary carriage-wheels, as well as the steel tyres for the wheels of locomotives and railway carriages, are fitted on when red-hot, and on cooling grip them with very great force. The same property of iron is utilized in the manufacture of the Armstrong gun. In these guns, the central tube used to be surrounded by a number of iron cylinders, made by winding bars into helices, and welding each turn to the next, so as to form a cylinder in which the fibre of the iron is arranged in a helix about the axis of the gun. The first cylinder was turned so as to fit easily over the breech of the gun, when the latter was cold but itself red-hot. It was then heated and slipped into its position, after which the gun was allowed to cool. On cooling, the cylinder tended to con-

tract, and exerted great pressure on the breech. The next cylinder was turned so as to fit over the first in a similar way; and so on, Armstrong guns sometimes having 5 or 6 cylinders placed one over the other. Each cylinder, except the outside one, was thus subjected to an enormous pressure, on account of the tendency of the cylinder surrounding it to contract upon it, while in its turn it exerted pressure on the cylinder within it. Hence, when the powder exploded, the pressure of the gases produced had to balance the contractile forces of all the cylinders before it could exert any force effective in producing a strain on the central tube of the gun. Guns thus constructed will consequently bear the explosion of very heavy charges without bursting; a pressure of about 70 tons' weight per square inch being sometimes exerted within them.

109. The employment of prismatic powder, which burns comparatively slowly, has very much reduced the maximum pressure to which heavy guns are subjected, while it has enabled a very considerable pressure to be sustained for a long time upon the shot, which is thus subject to nearly the full pressure over a considerable portion of the length of the very long barrels now employed; thus a great deal more work is done upon the shot with much less danger to the gun. The experiments of Sir Frederick Abel and Captain Noble on the explosion of powder in a closed vessel, showed that the maximum pressure exerted by the powder gases is about 48 tons on the square inch. In modern artillery the pressure developed does not generally exceed 20 tons on the square inch, and frequently does not much exceed 12 tons. The very high pressures which have been registered in old guns with fine grained powder appear to be partly due to wave action increasing the pressure as the wave of compression was transmitted through the gas.

Heavy guns now are built entirely of steel, which is finally tempered in oil. The several coils are shrunk on to the tube, the diameter to which each is turned being determined by calculation, so that the stress in the steel may be as nearly as possible uniform throughout the

whole structure when the powder explodes. This condition implies that the tension in the several coils should increase outwards, so that the outermost coil is subjected to the greatest tension.

110. The same principle may, with advantage, be applied to hydraulic cylinders, or other vessels exposed to bursting pressure, whenever the pressure is so great that it is difficult to make ordinary forgings or castings capable of sustaining it.

In several cases in which the walls of buildings have bulged outwards, they have been drawn together by passing iron bars through them across the building, heating the bars, and, when expanded, screwing nuts upon their projecting ends. On cooling, the bars have drawn the walls together, and the process has been repeated till the walls assumed their proper positions.

111. An idea of the force which iron exerts if prevented from contracting when cooled from a high temperature to ordinary temperatures, may be gathered from the following rough calculation.

Suppose an iron bar, one square inch in section, cooled from  $500^{\circ}\text{C}$ . (a dull red heat) to  $0^{\circ}\text{C}$ . It would, if allowed, contract from a length represented by 1.006 to a length represented by 1, its coefficient of expansion being about .000012. The bar therefore when at  $0^{\circ}\text{C}$ ., is stretched beyond its natural length by  $\frac{6}{1000}$  of that length. Now the force capable of stretching a bar of iron of 1 sq. inch section by this amount is about equal to the weight of 15 tons, which therefore represents the force with which it tends to contract. If heated to only  $100^{\circ}\text{C}$ ., the force it exerts on cooling will be equal to the weight of about 13 tons, the elasticity of iron being very imperfect when stretched so much.

112. **Precautions Necessitated by Expansion and Contraction.** Though the expansion of metals by heat is thus frequently applied to useful purposes, it is perhaps more frequently a source of trouble. Thus, in order to allow for expansion, the pipes of water and gas

mains have to be connected by telescope joints; the consecutive metals on a railway have to be placed at a small distance apart, the bolt-holes through which they are bolted to the fish plates being elongated; and so on. The tubular girders of the Britannia bridge are each mounted on rollers at the ends, while the metals are halved together, as shown in Fig. 16, so as to allow a play of about nine inches, without making a gap between them.

FIG. 16.



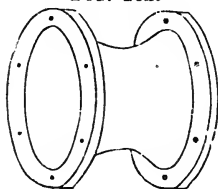
113. The patterns for castings must be made somewhat larger than the castings required. In the case of cast iron about an eighth of an inch per foot is allowed for contraction on cooling.

The contraction of metal castings when cooling often produces great strains in the metal, and not unfrequently the castings are broken. For example, in a wheel having a thin rim and thick round arms, and a massive boss, the rim will solidify and cool before the arms and boss. As the arms cool subsequently, the contraction of the arms and boss is opposed by the rim of the wheel, and one or more of the arms may be fractured. To avoid this the arms are often bent, so that their contraction on cooling may tend to straighten instead of fracturing them.

As castings are frequently broken through one portion cooling more slowly than another, it is clear that in many cases, though fracture may not be produced, stresses are caused in the castings which may be on the point of producing fracture. If such castings are loaded so as to increase these stresses ever so slightly, fracture may ensue though the load may be far less than the computed "safe load" for the structure. The strains induced in glass by rapid cooling will be referred to in Chapter VII.

114. If furnace bars are made with square ends so as to fit tightly in their places, they are compelled to bend on account of the expansion they undergo when heated. For this reason the ends of furnace bars should be bevelled off on the lower side at an angle less than the limiting angle of friction, and should be carried on bearers bevelled to the same angle, plenty of room being left between the ends of the bars and the ends of the furnace.

FIG. 16A.



115. The flanged iron pipes often used to convey steam from boilers to engines are sometimes connected by copper connecting pieces of the form shown in Fig. 16A. As the pipes expand or contract, these copper connections have their curvatures changed, so as to accommodate themselves to the pipes. "Expansion joints" in the form of telescopic slides are frequently employed in place of these connecting pieces, and often long bends, or "spring pipes," are used to obviate the necessity of expansion joints.

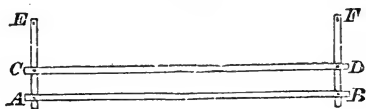
116. When roof principals of considerable span are constructed of iron, it is necessary to provide for their expansion without interfering with the walls. Such principals should rest on rollers carried on the wall plates, and fixed abutments should be placed at such a distance as to give plenty of room for expansion. The effect of wind on one side of the roof will then be to cause the principals to press against the abutments on the opposite side, and through these the horizontal wind pressure will be transmitted to the outside buttresses.

117. In the construction and use of standards of length, it is of extreme importance that the temperature should be carefully observed, and corrections made for expansion or contraction. The Imperial standard yard is defined as the distance between the centre of two fine lines engraved on the gold plugs which are inserted in a certain bronze bar kept in the Exchequer Chamber, and known as the Imperial standard yard, the temperature of the bar being  $62^{\circ}\text{F}$ . From this it will be seen that before employing the bar as a standard, it must be brought to the temperature of  $62^{\circ}\text{F}$ . The room at Southampton, which was constructed by Col. Clarke for the standards belonging to the Ordnance Survey, is most carefully protected from changes

of temperature by double walls and double doors, so arranged that the first must be closed before the second can be opened.

118. In field work it is of course impossible to keep the measuring bars always at the same temperature, and it is therefore necessary to know their coefficients of expansion, and to observe their temperature from time to time during the day, in order to make the necessary corrections. The measuring bars designed by Colonel Drummond for the Indian Survey are so arranged, or compensated, that the distance between the marks is independent of the temperature. Their construction is illustrated

FIG. 17.



by the diagram (Fig. 17).  $AB$  is a bar of brass, and  $CD$  an equal bar of iron. These are connected by the cross pieces  $ECA$ ,  $FDB$  by pins at  $A$ ,  $B$ ,  $C$  and  $D$ . The points which mark the ends of the standard are placed at  $E$  and  $F$ , so that  $AE : CE :: BF : DF :: \text{coefficient of expansion of brass} : \text{coefficient of expansion of iron}$ . If the temperature be raised  $AB$  expands more than  $CD$ , and the bars  $AE$ ,  $BF$  are caused to incline towards one another in such a way that the distance between  $E$  and  $F$  remains unchanged. Microscopes are employed along with the bars, and one microscope is placed upon the ground and its movable wire adjusted till it coincides with the image of  $E$ . The bar is then advanced, and so placed that the image of  $F$  is on the wire, and then the microscope is again moved to  $E$ . Two microscopes are generally employed to prevent accidents, and the first is not moved till the second is adjusted.

119. Perhaps the expansion of metals causes more trouble to horologists than to any other class of persons. The rate of a clock is usually governed by a pendulum, and the time of vibration of a pendulum depends on the distance between the centres of suspension and of oscillation. Now, if the pendulum consist of a simple metallic rod carrying a weight, when it is heated it expands, and this distance is increased, so that the clock is retarded, while on cooling it is accelerated. A compensating pen

dulum is one in which the distance between the centres of oscillation and suspension is independent of the temperature.

120. A compensating pendulum may be constructed on precisely the same principle as the Drummond bars, with this difference, that the cross piece  $BD$  (Fig. 17) should be rigidly connected to the bars  $AB$  and  $CD$ , so that only  $AE$  may change its inclination. The centre of suspension should be connected to  $B$ , the bar  $AB$  being somewhat produced, and the pendulum bob suspended from  $E$ . For the sake of symmetry the arrangement may be duplicated about  $AB$ , so that there are bars corresponding to  $CD$  and  $AE$  on each side of  $AB$ . Ellicott's pendulum is only a modification of this, the iron bar being placed between two brass bars, and a ring attached to the lower end of the iron bar, which is pivoted to two cross pieces, from the outside extremities of which the bob is suspended. The brass bars press upon the other extremities of the cross pieces, and thus by their expansion raise the points of suspension of the bob.

121. In Graham's mercurial pendulum the bob consists of a glass vessel containing mercury. Now the coefficient of apparent expansion of mercury in glass is very much greater than the coefficient of *linear* expansion of the iron

FIG. 18.



suspension-rod, and while the latter expands downwards, the centre of gravity of the mercury in the glass vessel is raised by the expansion of the mercury, and the quantity of mercury is so arranged that the latter expansion just compensates for the former, and the centre of oscillation remains unmoved. The rod of Graham's pendulum was made of glass, but mercurial pendulums are now usually constructed with iron rods.

122. Harrison's gridiron pendulum is shown in Fig. 18. The vertical rods there indicated by black lines consist of iron, those shown by light lines, of which only four are shown in the figure, though there should be at least six, are made of brass. It is obvious from the construction that the expansion of the iron rods, which are sup-



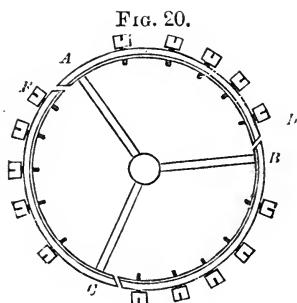
ported at their upper ends, tends to lower the bob, while that of the brass rods, all of which are supported at their lower ends, tends to raise it. Since the coefficient of expansion of brass is, roughly speaking, about  $1\frac{1}{2}$  times that of iron, it follows that the ratios of the lengths of the brass bars to those of the iron bars, and the distribution of metal throughout the pendulum, can be so arranged as to keep the distance between the centres of suspension and oscillation independent of the temperature.

123. A form of pendulum which is much more easily constructed than the gridiron pendulum, and answers better, inasmuch as it exposes less surface to the friction of the air, and has its centre of mass much lower down, is shown in Fig. 19. It consists of an iron tube  $AB$ , shown in section in the figure; inside this tube, and fitting it loosely, is placed a zinc tube, which is supported by being rivetted to the iron at  $B$ . From the top of the zinc tube hangs an iron rod  $CD$ , which carries the bob. Here it is obvious that the pendulum will be compensated if  $BC$  be about equal to  $\frac{2}{5} (AB + CD)$ , since the coefficient of linear expansion of zinc is about  $2\frac{1}{2}$  times that of iron. In order to give sufficient length to the zinc tube, these pendulums are usually constructed so that the point  $B$  is within a hole drilled through the bob.

FIG. 19.



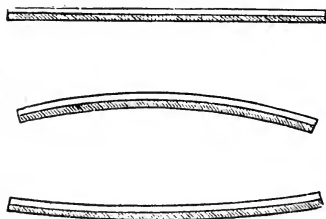
124. The movements of watches and chronometers are governed by "balance-wheels" which oscillate under the action of the elasticity of the hair-spring, to which they are attached. The time of oscillation of the wheel depends upon its moment of inertia, and increases with it. Now this is increased by increasing the diameter of the wheel, though the mass remains unchanged. Hence a simple balance-wheel will cause the rate of the watch to diminish as the temperature increases. This is obviated in the chronometer balance-wheel shown in Fig. 20,



by making the circumference of the wheel of two metals, the outer of which is some metal, as brass, whose coefficient of expansion is great, while the inner metal is steel, or some other, whose coefficient of expansion is small. The circumference is cut into two or three parts, each of which is supported at one end by an arm of the wheel. Now, if the temperature increase, the distances of the points *A, B, C* from the centre of the wheel will increase on account of the expansion of the arms, while the curvature of the arcs *AD, BE, CF* will increase because the outer metal will expand more than the inner. This will occasion the points *D, E, F* to approach the centre, and the weights of the heavy screws attached to each arc can be so adjusted that the moment of inertia, and therefore the time of oscillation of the wheel, is unaffected by change of temperature. Balance-wheels are generally made by first turning a small wheel in steel and then casting a ring of brass around it before cutting through the circumference.

Inasmuch as the elasticity of the hair-spring is generally affected by change of temperature, diminishing as the temperature is raised, it follows that if the moment of inertia of the balance-wheel were kept constant the rate of the watch would diminish with increase of temperature on account of the change in the hair-spring. Hence it is necessary that the moment of inertia of the wheel should be diminished as the temperature is raised; that is, that

FIG. 21.



the balance-wheel should be over-compensated.

125. The change of curvature produced in a compound bar, similar to the circumference of the balance-wheel, by change of temperature, may be shown by rivetting

together at an ordinary temperature plane strips of zinc and brass, or of silver and platinum, as in Fig. 21, where the brass is shaded. On heating the strip it will assume the second form shown, while on cooling it in a freezing mixture it will take the third form. This experiment shows conspicuously the difference in the expansion of two metals for the same increments of temperature.

126. Breguet's thermometer consists of very thin strips of silver, gold, and platinum, which are rolled together so as to form a thin narrow ribbon, and this is wound into a helix, in which the silver is on the outside and the platinum on the inside. If the temperature of the ribbon increase, the helix winds itself tighter, because the expansion of the outside is greater than that of its inner surface; while, when it is cooled, it partially unwinds. If one end of the ribbon be fixed, and a needle or mirror attached to the other end, a very sensitive thermometer is obtained.

This thermometer is sometimes employed as a rough galvanometer for indicating the passage of an electric current. The helix is supported at the top by a metal arm connected with one electrode, while a wire connected with its lower end dips into a cup of mercury in communication with the other electrode. The current thus flows through the helix itself and manifests its presence by its heating effect.

127. The points upon railways are connected with the pointsman's lever by iron rods, and when the distance is considerable the expansion and contraction of the rods through changes of temperature would cause the points to move without any change in the position of the lever if no precaution were taken to prevent it. In order to overcome this difficulty the connecting rods are fastened to short levers *BCD*, *EFG* (Fig. 22), which can turn freely about pivots *C*, *F*, which are fixed relative to the per-

FIG. 22.



manent way. If  $A$  represent the end of the rod which is attached to the lever, and  $H$  the end which is connected with the points, the condition that  $H$  should remain fixed as long as  $A$  is fixed is that the expansion of  $DE$  should be equal to that of  $AB$  and  $GH$  together, or that the length of  $DE$  should be equal to the sum of the lengths of  $AB$  and  $GH$ . Any number of levers similar to  $BCD$  may be introduced, but the condition which must always be fulfilled by the bars is that the sums of the lengths of the alternate bars should be the same.

## CHAPTER V.

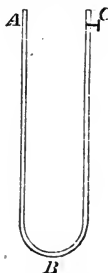
ON THE EFFECTS OF HEAT UPON GASES. THE GASEOUS LAWS. COOLING OF GASES BY EXPANSION. SPECIFIC HEAT OF AIR AT CONSTANT PRESSURE AND AT CONSTANT VOLUME. DIFFUSION OF GASES.

128. **Boyle's Law.** We have now to consider the effects of heat upon gases. A gas has been defined as *a fluid, a finite portion of which can be made to occupy any assigned space, however great, by sufficiently diminishing the pressure to which it is exposed.* In speaking, then, of the volume of a quantity of gas, it is of primary importance to specify the pressure under which this volume is measured. The law connecting the volume and pressure of a given mass of gas at constant temperature is known as Boyle's or Mariotte's, law, and is as follows:—

*The volume of a given mass of gas at constant temperature varies inversely as its pressure; or, The product of the numerical measures of the volume and pressure of a given mass of gas at constant temperature is constant.*

129. This may be shown experimentally by partially filling a tall U tube *ABC*, with mercury, closing the end *C*, and then pouring mercury into or out of the arm *AB*. It will be found that when the air in *BC* has acquired its original temperature after this operation, its volume will vary inversely as the sum or difference of the height of the mercurial barometer and of the difference in height of the surfaces of the mercury in the two arms of the tube,—that is, inversely as the whole pressure to which it is subjected; the difference of level

FIG. 23.



of the mercury in the arms being added to the height of the barometer when the mercury in  $AB$  stands above that in  $BC$ , and subtracted from it when the opposite is the case.

130. Instead of the  $U$  tube, a more convenient apparatus consists of two glass tubes connected by a few feet of flexible india-rubber tubing, having a small bore and thick wall, so as to be able to sustain a pressure of six or eight feet or more of mercury. The tube which corresponds to the arm  $C$  in Fig. 23, and which should be of uniform bore, or be calibrated, is fixed to a long vertical scale, while the tube corresponding to the arm  $A$  can be moved up and down the scale. The two tubes being placed at the same height, mercury is poured into the apparatus till it reaches the middle of the tubes. The tube  $A$  should then be raised till  $C$  is completely filled with mercury. The upper end of  $C$ , which has been left open, should then be connected with a desiccating apparatus, so that on lowering  $A$  pure dry air may be drawn into  $C$ , and when  $C$  is about one-third full of air its upper end should be sealed with the blow-pipe flame. The pressure upon the air in  $C$  may then be increased or diminished to any extent within the limits fixed by the length of the india-rubber tube, while the difference in level of the surfaces of the mercury in the tubes is measured by the scale. By employing a scale etched on plate glass and silvered at the back, all danger of parallax can be removed by causing the surface of the mercury to exactly cover its image formed in the mirror-scale. This last device is due to Professor Jolly.

131. **Expansion of air and other Gases.** The increase in volume of a quantity of air for a given change of temperature, when the pressure is kept constant, may be roughly determined by taking an empty thermometer tube, warming it, and, when it begins to cool, dipping the end under the surface of mercury for a very short time, so that a small pellet of mercury may be made to enter the

FIG. 24.



tube. The volume of the air within the instrument will then be determined by the position of the pellet of mercury. This apparatus will serve as an air thermometer for a small range of temperature. By heating the bulb in a bath, whose temperature is measured by a mercurial thermometer, it will be found that the volume of the air increases very nearly uniformly *for equal increments of temperature indicated by the mercurial thermometer*. It is not easy, however, to obtain accurate results with this instrument, as the pellet of mercury is apt to stick in the tube and then suddenly go forward with a rush.

132. If the thermometer be filled with any other gas, which cannot be readily liquefied, as oxygen, hydrogen, nitrogen, etc., it will be found that the coefficient of expansion is almost precisely the same as for *dry* air, and that for every increase in temperature of  $1^{\circ}\text{C.}$ , under any constant pressure, each of these gases expands by about

$\frac{1}{273}$  of the volume it would have at  $0^{\circ}\text{C.}$  under the same

pressure (though this rate is not quite uniform if the temperature be indicated by the mercurial thermometer). If the volumes of quantities of these different gases, always subject to the same pressure, be equal at some particular temperature, they will also be almost exactly equal when they are all at any other, the same, temperature. Hence, thermometers filled with any of the so-called permanent gases all indicate *the same scale of temperature*, which, however, as stated in Chapter I., differs very little from that of the mercurial thermometer.

133. **Charles' Law.** From the preceding we see that if  $v_0$  represent the volume of any quantity of a permanent gas at  $0^{\circ}\text{C.}$ , and  $v_t$  its volume under the same pressure at  $t^{\circ}\text{C.}$ , then

$$v_t = v_0 \left( 1 + \frac{1}{273}t \right).$$

Thus if the gas occupied 273 cubic inches at  $0^{\circ}\text{C.}$ , it would occupy 274 cubic inches at  $1^{\circ}\text{C.}$ , and 373 cubic inches at  $100^{\circ}\text{C.}$  The fraction  $\frac{1}{273}$  is often denoted by  $\alpha$ .

134. Now suppose that instead of having the air in a bulb tube we have it in a tube of uniform bore throughout, and closed at the lower end. Then the volume of the air will be proportional to the distance of the bottom of the pellet of mercury from the bottom of the tube. Suppose that when the temperature is  $0^{\circ}\text{C}$ . the air occupies 7.23 inches of the tube. Let a mark be placed at this distance from the bottom, and called the freezing point. Now place the whole tube in the steam above boiling water, as in the case of the mercurial thermometer described in Chapter I.; it will then be found that the air will occupy 37.3 inches of the tube. Marking the tube at this height (calling it the boiling point), and dividing the distance between this point and the freezing point into 100 equal parts, or degrees, each degree will occupy one-tenth of an inch of the tube. Marking off distances each equal to the tenth of an inch below the freezing point, and indicating them by the corresponding number with a negative sign prefixed, we find that the bottom of the tube is marked  $-273^{\circ}$ . Hence we infer that if the law of contraction of air with decrease of temperature remained the same as at ordinary temperatures, and we could cool the air to a temperature corresponding to  $-273^{\circ}\text{C}$ ., its volume would be zero. Of course we never expect to realize this, for even supposing it possible to attain this low temperature, which we never hope to do, the air would probably cease to be a gas long before reaching it. The temperature corresponding to the bottom of the tube is called the *absolute zero* of the air thermometer, and temperatures reckoned from this point are called *absolute temperatures*. The absolute temperature of the freezing point is therefore  $273^{\circ}$ , and that of the boiling point  $373^{\circ}$ . Now the volume of the air in the tube at any temperature is proportional to the length of the tube which it occupies, and therefore to the absolute temperature indicated by its upper surface. Hence we see that,

*The volume of any mass of air, or other gas, at constant pressure increases uniformly with the temperature, and is proportional to the temperature reckoned from the absolute zero of the air thermometer.*



This is sometimes called Charles', but more generally Gay Lussac's, law. Boyle's law and this together constitute "the gaseous laws," which should be carefully remembered.

135. Ex. *A quantity of gas occupies 10 cubic inches at a temperature of  $15^{\circ}\text{C}$ .; what will be its volume under the same pressure at  $75^{\circ}\text{C}$ .?*

$15^{\circ}\text{C}$ . corresponds to  $(273 + 15)^{\circ}$ , or  $288^{\circ}$  above the absolute zero of the air thermometer, while  $75^{\circ}\text{C}$ . corresponds to an absolute temperature of  $348^{\circ}$ .

At  $288^{\circ}$  Abs. Temp. the gas occupies 10 cubic inches.

$\therefore$  at  $1^{\circ}$  ... it would occupy  $\frac{10}{288}$  cubic ins.

$\therefore$  at  $348^{\circ}$  ...  $\frac{10 \times 348}{288}$  cubic ins.

$= 12.08$  ... cubic inches.

This is perhaps the simplest method of finding the relative volumes of a quantity of gas, corresponding to different temperatures.

136. **Relation between Pressure and Temperature at Constant Volume.** Boyle's law states that the product of the numerical measures of the pressure and volume of a given mass of gas remains constant while the temperature remains so; and we have just seen that if the pressure be kept constant the volume is proportional to the absolute temperature.

Now, let  $v$  denote the volume and  $p$  the pressure of a quantity of gas whose temperature, reckoned from absolute zero, is  $T^{\circ}$ . Let the temperature be changed to  $T_1^{\circ}$ , the pressure remaining the same. Let the volume then become  $v_1$ . Then

$$v_1 = v \frac{T_1}{T} \dots \dots \dots (1),$$

$$\text{and } pv_1 = pv \frac{T_1}{T}.$$

Now let the pressure be increased, the temperature remaining at  $T_1^{\circ}$  till the volume is again reduced to  $v$ , and let  $p_1$  denote the pressure when this is the case. Then, since the temperature is kept constant,

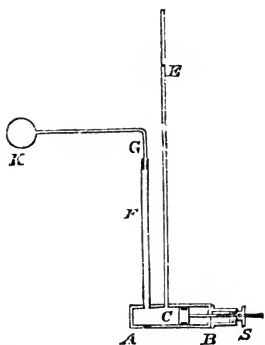
$$p_1 v = pv_1, \text{ by Boyle's law.}$$

$$\begin{aligned}\therefore p_1 &= p \frac{v_1}{v} \\ &= p \frac{T_1}{T} \text{ by (1).}\end{aligned}$$

Now, if the gas were heated to  $T_1^\circ$ , its volume being kept constant, we should finally have arrived at the same state of things as by the process we have adopted, and the pressure would then, as now, be denoted by  $p_1$ . Hence,

*The pressure of any mass of gas whose volume is kept constant increases uniformly with the temperature, and is proportional to the temperature reckoned from the absolute zero of the air thermometer.*

FIG. 25.



137. This result may be tested experimentally by an apparatus designed by Professor Balfour Stewart, the essential portions of which are shown in Fig. 25.

$AB$  is a vessel containing mercury, and closed at one end by a piston, which can be made to slide in and out through a stuffing box by means of the screw  $S$ , and thus to change the capacity of the vessel  $AB$ .  $E$  is a vertical glass tube communicating with the interior of  $AB$ , and open at the top, while the tube  $F$  is connected with a very fine tube leading from the glass bulb  $K$ . The bulb  $K$  is filled with perfectly dry air, and then attached to  $F$ .  $K$  is then cooled to some known temperature, as by placing it in a quantity of melting ice, and the mercury in  $F$  is made to rise to a marked point  $G$ , where the tube is very narrow, by turning the screw  $S$ . The excess of the height of the mercury in the open tube  $E$  over its height in  $F$ , when added to the height of the barometer, gives the whole pressure to which the air in  $K$  is subjected. Call this  $P$ . The bulb  $K$  is then heated to some known temperature, as, for instance, by placing it in the steam over boiling water, and the piston  $C$  is advanced by means of the screw  $S$

till the mercury in  $F$  again reaches  $G$ . Determining as before the pressure to which the air in  $K$  is subjected, and denoting it by  $P_1$ , we find that

$$P_1 = P \frac{T_1}{T},$$

where  $T$  and  $T_1$  denote the temperatures of the bulb in the first and second experiments respectively, reckoned from the zero of the air thermometer. If the temperatures employed are those of melting ice and of the steam over water boiling at the standard pressure, we find

$$P_1 = P \frac{373}{273}.$$

The cylinder and piston, which is necessarily a somewhat expensive apparatus, may be replaced by a strong vessel of india-rubber, which can be slowly compressed between two plates of metal by means of a screw. The apparatus is rendered much more convenient for use by bending the tube  $KG$  so that it may rise vertically for an inch or two from the bulb. It is then easier to immerse the bulb in a vessel of water.

138. Professor Jolly's air thermometer, which serves the same purpose as the last-mentioned instrument, is similar to the apparatus described in Art. 130, except that the tube  $G$  is replaced by a bulb tube so bent that it can be conveniently raised to any desired temperature in a bath, and having a portion of its neck, in which the surface of the mercury is kept, very narrow. The bulb tube is also supplied with a branch and stop-cock, through which it can be exhausted and filled with any gas which may be desired.

139. **Summary of the Gaseous Laws.** We have seen then, that, if the pressure be kept constant, the volume of a given mass of gas varies as its absolute temperature; while if the volume be kept constant, the pressure varies as the absolute temperature. Hence, when both volume and pressure are allowed to vary together,

*The product of the numerical measures of the pressure and volume of a given mass of gas increases uniformly with the temperature, and is proportional to the temperature reckoned from the absolute zero of the air thermometer.*

140. As an example of the application of the preceding laws, we may take the following:—

*A quantity of air occupies 1 cubic foot at a temperature of 59° F. and under a pressure of 30·24 inches of mercury. It is required to find its volume at 113° F. under a pressure of 28·62 inches of mercury.*

59° F. corresponds to 15° C., or an absolute temperature of 288°; while 113° F. corresponds to 45° C., or 318° C. above the absolute zero of the air thermometer. Hence, if the pressure remained constant, the volume at 113° F. would be

$$1 \times \frac{318}{288} \text{ cub. feet.}$$

But the pressure changes from 30·24 to 28·62 inches of mercury, and the volume varies inversely as the pressure. Therefore the actual volume is

$$\frac{318}{288} \times \frac{30\cdot24}{28\cdot62} \text{ cub. ft.} = 1\frac{1}{6} \text{ cub. ft.}$$

Or we may proceed thus:—

Volume at 288° under pressure of 30·24 inches is 1 c. ft.

$$\therefore \dots \dots 1^\circ \dots \dots 30\cdot24 \dots \dots \frac{1}{288} \text{ c. ft.}$$

$$\therefore \dots \dots 1^\circ \dots \dots 1 \text{ inch} \dots \dots \frac{30\cdot24}{288} \text{ c. ft.}$$

$$\therefore \dots \dots 1^\circ \dots \dots 28\cdot62 \text{ inches} \dots \dots \frac{30\cdot24}{288 \times 28\cdot62} \text{ c. ft.}$$

$$\therefore \dots \dots 318^\circ \dots \dots 28\cdot62 \dots \dots \frac{20\cdot24 \times 318}{288 \times 28\cdot62} \text{ c. ft.}$$

141. **Absolute Zero.** We have seen that if the volume be kept constant, the pressure of a given mass of gas is proportional to its temperature above the absolute zero of the air thermometer. Hence, at the absolute zero of temperature the pressure of any quantity of gas would be zero, supposing the same law to hold at such a low temperature. Now, according to the molecular theory of gases, a gas consists of a number of very small particles or molecules, moving about with great velocity in all

directions (Art. 86). The pressure of the gas on any surface in contact with it is due to the continual impacts of these molecules; and it may be shown that the pressure of a gas upon a surface is proportional to the kinetic energy possessed by the unit of volume of the gas. Hence, when this pressure is zero, it follows that the gas possesses no kinetic energy, and that its particles are therefore at rest. This is a reason why the zero of the air thermometer should be considered to be the absolute zero of temperature.

**142. Coefficients of Expansion of Gases.** A *perfect gas* is an ideal substance which strictly follows Boyle's law. All known gases deviate to some extent from this law, and those which deviate most from Boyle's law also deviate most from Gay Lussac's law. Gases which can be readily liquefied deviate considerably from the gaseous laws, and the more so as they approach the temperature and pressure at which they become liquid, when the coefficient of expansion rapidly increases.

As a general rule gases have a greater coefficient of expansion than liquids, and liquids than solids. The coefficients of expansion of liquids and solids generally increase as the temperature is raised, if the temperature be measured by the air thermometer.

**143.** In the following table the second column contains the mean coefficients of expansion of the gases mentioned in the first column between  $0^{\circ}\text{C.}$  and  $100^{\circ}\text{C.}$

Name of gas.	Constant pressure.	Constant volume.
Hydrogen	·003661	·003667
Air	·003670	·003665
Nitrogen		·003668
Carbonic oxide	·003669	·003667
„ anhydride	·003710	·003688
Nitrous oxide	·003719	·003676
Sulphurous anhydride	·003903	·003345
Cyanogen	·003877	·003829

at constant ordinary pressure, and the third column indicates the mean rate of increase of pressure of the same gases for the same range of temperature when the volume is constant. The numbers are due to Regnault.

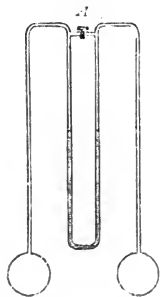
144. From the table given in the last Article it will be seen that the coefficients for hydrogen, nitrogen, and carbonic oxide, gases which can be liquefied only by means of intense cold and pressure, are sensibly the same as for air; while in the case of each of these gases the rate of increase of volume per degree of temperature, when the pressure is constant, is very nearly the same as the rate of increase of pressure when the volume is constant; or the gases obey Boyle's law. The coefficients of expansion of the other gases mentioned in the table differ sensibly from that of air, and in their cases the rates of increase of volume and of pressure are not the same; that is, the gases diverge sensibly from Boyle's law. These gases can be liquefied with comparative ease.

As mentioned in Chapter I., it is the almost exact agreement between the expansions of air, hydrogen, oxygen, nitrogen, carbonic oxide, and some other gases at different temperatures, which renders the scale of the air thermometer of peculiar importance; and, moreover, the specific heat of these gases is the same at all temperatures measured by the air thermometer.

#### 145. Differential Thermometer.

The great coefficient of expansion of air renders it useful for the construction of sensitive thermometers. A common form of the differential thermometer is shown in Fig. 26. It consists of a glass tube bent as shown in the figure, and having a bulb at each extremity. The middle portion of the tube is partly filled with coloured liquid, while the two upper bends are united by a short tube which can be closed by a plug tap *A*. When *A* is opened the pressures in the two bulbs are equalised, and the liquids stand at the same height in each limb. The tap *A* is then closed. Now, if the tem-

FIG. 26.



each limb. The tap *A* is then closed. Now, if the tem-

perature of the bulbs be initially the same, and if the temperature of either, or both, change, the pressure of the air will be greater in that whose temperature suffers the greater increase, and the liquid will be driven by the excess of pressure towards the other bulb, the difference of the heights at which it stands in the two limbs roughly indicating the difference in the changes of temperature of the two bulbs. If the bulbs differ in temperature when  $A$  is closed, the liquid will remain undisturbed provided the absolute temperature of each be increased by a quantity proportional to itself.

Leslie's differential thermometer was somewhat simpler than that just described, the tube being bent only twice at right angles, and a bulb being placed at the top of each branch of the tube.

**146. Air heated by Compression.** If a quantity of air be confined in a cylinder into which a piston is accurately fitted, and the piston be suddenly and violently depressed so as to compress the air, the latter will be considerably heated; so much so that German tinder placed within the cylinder may be ignited, or if a glass cylinder be used, and a little vapour of bisulphide of carbon diffused through the air within it, the vapour may be inflamed, producing a brilliant flash.

**147.** Suppose a quantity of air to be heated from  $T^{\circ}$  to  $T_1^{\circ}$  in a closed vessel, so that its volume is kept constant. A certain amount of heat will be absorbed. Now let an equal quantity of air be heated through the same range of temperature, its pressure being kept constant, and the air therefore allowed to expand. The two quantities of air have received the same increment of temperature, but under very different circumstances. In the second case, work has been done against the external pressure by the expanding air; in the case of the air being heated in a closed vessel no such external work is done. We cannot, however, at once conclude that more heat has been absorbed in the latter case than in the former, because the air at the end of the operation is not in the same condition in the two cases. Joule, however, showed that when air expands into a vacuum, so as to do

no external work, there is no appreciable amount of heat absorbed or liberated, so that no heat is required simply to change the volume of the air. The same is the case with all the more perfect gases, but cannot be said of other bodies. As far, therefore, as the heating of the air is concerned, the same amount of heat will be required to raise the temperature of the air from  $T^{\circ}$  to  $T_1^{\circ}$  whether the volume or the pressure be kept constant; but in the latter case external work is done, the energy corresponding to which must be supplied from some source, and the only source available is the heat supplied. In fact, if air be allowed to expand and do external work without receiving heat, its temperature falls, because the energy corresponding to the work done has to be derived from the heat of the gas itself. We therefore require more heat to raise a quantity of air through a certain number of degrees at constant pressure, than is necessary when the volume is kept constant by an amount equivalent to the external work done by the air in expanding. From this experiment we learn that the *specific heat of air at constant pressure* is greater than its *specific heat at constant volume*. If both the pressure and volume were allowed to increase, the specific heat would be intermediate between these two; while if the volume increased so much that the pressure diminished, it would be greater than the first; and if the pressure increased so much that the volume diminished, it would be less than the second. We shall see that the kinetic theory of gases completely explains the phenomena referred to.

**148. Cooling of Air by Expansion.** Since gases become heated when suddenly compressed, it will follow that, if allowed to expand by relieving the pressure, the gases will be cooled, provided they do not receive heat from without. We shall meet with instances of this in a subsequent chapter.

The low temperature produced by air when expanding and doing work is very noticeable in engines driven by compressed air. The exhaust air escaping from the cylinder will sometimes produce a shower of snow, if the external air be warm and not very dry, while the exhaust



pipe and valves become coated with hoar frost. The "air-compressors" or force pumps, by which the air is compressed previously to being transmitted to the motors, or compressed air engines, require very special arrangements to keep them cool, since all the work done by the steam engine, or other source of power, in compressing the air, is converted into heat in the air as it is compressed. When very high pressures, such as 400 lbs. or 500 lbs. on the square inch, are employed, the air is compressed in two successive operations in different pumps, the compressed air from the first pump being operated upon and further compressed by the second, while water is made to circulate round the barrels of both pumps.

The expansion of compressed air is frequently employed for the production of very low temperatures, as, for example, in freezing meat for conveyance from Australia and America to this country. The air is compressed, and the compressed air cooled to the ordinary temperature. It is then allowed to expand, and is thereby cooled to a temperature far below the freezing point. It then passes through the chamber containing the meat, and cools the chamber and contents down to the temperature required for the voyage. The work done by the compressed air in expanding may of course be utilized for any purpose. In particular, it might be employed in *assisting* to work the compressing pumps.

149. If a quantity of gas be allowed to expand without doing any external work, as when it expands into a vacuum, no heat is lost or gained by the gas as a whole. This was shown by Dr. Joule, who connected a vessel containing compressed air, at a pressure of about 22 atmospheres, with a second vessel which was exhausted, the communication between them being closed by a tap. The two vessels were placed under the water in a calorimeter, and after they had acquired the temperature of the water, the tap was opened. The temperature of the water remained unaffected. Hence, when air expands without doing external work, no heat is lost or gained by it. The air left in the first vessel did work in driving the air into the second with great velocity, and was itself

cooled, but the air entering the second vessel on striking against its sides acquired as much heat by the impact as was lost by that in the first, so that as much heat was communicated to the water by one vessel as was taken from it by the other.

**150. Regnault's Laws.** In Art. 48 we explained how the specific heats of air, and other gases, at constant pressure, were determined by Regnault. The results of his investigations may be summed up in the following laws.

(1) *The specific heat of a permanent gas at constant pressure is independent of that pressure, and the same for all temperatures.*

(2) *The capacity for heat of EQUAL VOLUMES of different permanent gases at the same pressure and temperature are equal.*

(3) *The specific heats of the easily condensible gases vary slightly with the temperature.*

The experimental determination of the specific heat of air at constant volume is one of extreme difficulty. An indirect method by which this has been effected will be described in Chapter X. We may, however, here remark that, for all the more perfect gases, the ratio of the specific heat at constant pressure to that at constant volume is independent of the temperature of the gas, and for air is about 1.41.

**151. The Air and the Mercurial Thermometers.** It has been stated, Art. 131, that the scale of the mercurial thermometer differs but very slightly from that of the air thermometer. In the course of his experiments on the maximum pressure of aqueous vapours at different temperatures, Regnault made a series of comparisons between the readings of his mercurial thermometer with an envelope of crown glass, and those of an air thermometer. Some of his results are given in the following table. Below  $100^{\circ}$  C. the readings were practically the same.

Air Thermometer.	Mercury Thermometer.
100° C.	100° C.
140	139·85
180	179·63
200	199·70
240	239·90
260	260·20
300	301·08
340	343·00
350	354·00

152. **Pyrometers.** The expansion of gases has been turned to practical account in the construction of pyrometers, or instruments for measuring very high temperatures, as of furnaces. Of these Regnault's mercury pyrometer may be taken as an example. It consists of a bottle of iron, or porcelain, the neck of which has a narrow opening, and is covered by a lid whose diameter is considerably greater than its own. The lid has a small hole bored through it, and the top of the neck and lower surface of the lid are ground together, so that by sliding the lid on the neck the bottle may be opened or closed. A small quantity of mercury is placed in the bottle, and the whole placed in the furnace. The quantity of mercury must be sufficient to expel, when volatilized, all the air from the bottle, and when it has acquired the temperature of the furnace, the neck is closed by sliding the cover, and the instrument removed and allowed to cool. When cold the mercury within it is weighed, and this gives the weight of mercury vapour which filled the bottle at the temperature of the furnace. The capacity of the bottle is determined by weighing it when full of mercury, and weighing it when empty. Then if we know the height of the barometer, and the specific gravity of mercury and of mercury vapour at some temperature and pressure, as well as the law of expansion of mercury vapour, we can calculate the temperature of the furnace.

153. Suppose, for example, that the amount of liquid mercury at 15° C. required to fill the bottle is 314 ozs., and that at this temperature a cubic foot of mercury weighs 13,564·8 ozs. Then it follows that the capacity of

the flask is 40 cubic inches. Suppose also that the amount of mercury left in the bottle after being removed from the furnace is 13 grains, and that at  $360^{\circ}\text{C}$ . a cubic foot of mercury vapour weighs 1580 grains, when under the same pressure as during the experiment. Then, if the temperature of the furnace be  $T^{\circ}\text{C}$ ., 40 cubic inches of mercury vapour will weigh

$$1580 \times \frac{40}{1728} \times \frac{360 + 273}{T + 273} \text{ grains,}$$

supposing mercury vapour to behave as a perfect gas at temperatures above  $360^{\circ}\text{C}$ . But

$$1580 \times \frac{40}{1728} \times \frac{360 + 273}{T + 273} = 13,$$

therefore 
$$T + 273 = \frac{1580 \times 40 \times (360 + 273)}{1728 \times 13}$$

$$= 1780.87\dots$$

therefore 
$$T = 1507.87\dots$$

and the temperature of the furnace is nearly  $1508^{\circ}\text{C}$ .

154. In Wedgwood's pyrometer the temperature is measured by the contraction of a piece of baked clay; others have been constructed in which the temperature is indicated by the expansion of a platinum bar. The action of Siemens' pyrometer depends upon the increase of the resistance which platinum wire offers to the passage of an electric current as its temperature is raised. It consists of a very fine platinum wire wound on a small porcelain bobbin, the ends of the wire being attached to very thick wires, whose resistance is exceedingly small compared with its own. The bobbin is placed within the furnace, with the thick wires leading from it to any convenient place, and the resistance of the wire is then measured in the usual way, and the temperature of the furnace deduced.

155. In cases where a large instrument can be employed, a convenient pyrometer consists of a vessel containing air to which is attached a pressure gauge similar to an ordinary steam gauge. The pressure of the air is registered by the gauge, and as its volume is nearly constant, its absolute temperature is nearly proportional to its pressure,

The communication between the heated air vessel and the pressure gauge must be made by means of a very fine tube, the capacity of which is very small compared with that of the heated vessel, while its length is sufficient to protect the gauge from the heat.

Murrie's pyrometer depends on the increase of the vapour pressure of a liquid as its temperature is raised, the pressure being measured by an ordinary gauge, as in the instrument just referred to. The liquid generally employed is mercury, which is contained in a steel vessel. This vessel communicates with the gauge by a fine steel pipe, which dips below the surface of the mercury, and which is filled with mercury to a considerable height. Between the mercury and the gauge there is a small quantity of air to protect the gauge from the action of the mercury. The upper portion of the mercury in the steel tube never attains a high temperature, but serves simply to transmit the pressure from the heated vessel. To prevent the vessel expanding under the high pressure to which it is exposed, especially when heated nearly to redness, it is enclosed within a second vessel, the space between being nearly filled with mercury, so that the pressure of the mercury vapour on the outside of the inner vessel may balance that on the inside. It may be observed that it is not necessary to actually generate any sensible amount of vapour within the inner vessel in order to raise the pressure to that corresponding to the temperature.

**156. • Diffusion of Gases.** If two vessels containing different gases be allowed to communicate with each other for some time, it will be found that each vessel contains a mixture of the gases of the same composition, and this will be the case even if one of the gases be much denser than the other, and the vessel containing it be placed at a considerable distance below that containing the lighter gas, the communication being made by a tube. Oxygen may be used for the heavier, and hydrogen for the lighter gas, and the mixture subsequently exploded.

This phenomenon is known as the *diffusion of gases*, and in virtue of it the atmosphere has nearly the same composition at all heights to which men have ascended. It

is sometimes expressed by saying that one gas acts as a vacuum to another.

157. If, in accordance with the molecular theory of gases, we suppose that a gas consists of a number of very small particles moving in all directions with great velocities, we see that they will penetrate into any space which is open to them, and the presence of any other gas in that space will only delay their diffusion through it by causing them to continually come into collision with its own particles, just as the passage of a person through a crowd is impeded by the presence of others, and would be much more so were all the persons composing the crowd, and including the intruder, highly elastic and perfectly devoid of intelligence. The molecular theory of gases thus readily accounts for the phenomenon of diffusion, and we see that either gas will ultimately be uniformly distributed throughout each vessel, because it is only then that the number of its particles passing from the first to the second vessel in any given time will be equal to the number passing in the opposite direction.

The molecular theory of gases also teaches us that at the same temperature the mean squares of the velocities of the particles of different gases under any given pressure vary inversely as the density of the gas; and this accounts for the law discovered by Graham, that gases diffuse into one another at rates varying inversely as the square roots of their densities.

158. If two streams of gas are moving in opposite directions, or in the same direction with different velocities, and are passing one another, particles will leave each stream and enter the other, and thus communicate to each stream some portion of the momentum of the other stream. The action is similar to that which would take place if two trains were passing one another, and a constant interchange of passengers took place between them, the passengers in each stepping into the other and back again. It is obvious that this action will tend to bring the trains or the streams of gases to relative rest; that is, to change the motion of each until the two are moving in precisely the same manner. This is the diffusion of momentum,

and completely explains the viscosity, or apparent friction of gases. It is obvious that the greater the molecular velocities of the gases the more rapidly will the particles of the two streams diffuse into each other, and the more rapid will be the diffusion of momentum. Hence, the viscosity of gases increases rapidly with the temperature, and at very high temperatures a quantity of gas may be so viscous as almost to resemble treacle in its character. This increase of viscosity with increase of temperature must be borne in mind in any calculations on the flow of furnace gases through boiler tubes, flues, or chimneys.

**159. Fire-Damp Indicators.** The differences in the rates of diffusion of different gases have been employed to detect the presence of coal-gas or fire-damp. An instrument for this purpose, known as Ansell's fire-damp indicator, consists of a small balloon made of thin india-rubber, and inflated with air. A linen band placed round its equator necessitates any expansion it may undergo taking place in the direction of its vertical diameter. An alarum is so placed that if the balloon, which is fixed into a suitable frame, expand vertically it will, by raising a detent, start the alarum. If the apparatus be placed in an atmosphere containing hydrogen or carburetted hydrogen, more gas will enter the balloon by diffusion through the envelope than will leave it, because at first it contains only pure air, and both oxygen and nitrogen are more dense than hydrogen or carburetted hydrogen. The contents of the balloon being increased, the envelope will expand vertically and liberate the alarum.

**160.** Another convenient form of fire-damp indicator consists of a helix of thin platinum wire, heated to dull redness by an electric current. If fire-damp or other combustible gas so present, it will burn *on the surface* of the platinum wire and raise its temperature. If the same current of electricity be sent through a second helix of platinum precisely similar to the first, but protected from the fire-damp, the amount of light given out by the helices may be compared as in an ordinary photometer, and the comparison serves to determine the percentage of fire-damp present.

## CHAPTER VI.

ON THE EFFECTS OF HEAT IN PRODUCING CHANGES OF STATE  
IN BODIES. LATENT HEAT. REGELATION. EVAPORATION.  
BOILING. DEW-POINT. CRITICAL POINT. SPHEROIDAL  
STATE.

161. **Liquefaction.** If heat be made to enter a solid body its temperature will, in general, be increased, and the solid will expand, until it arrives at a certain temperature (which depends for each solid only on the pressure to which it is subjected) called the fusing point, when the solid will begin to liquefy, and the temperature will remain unchanged till the whole of the solid is converted into the liquid state. During this process the volumes of most bodies increase, but there are many substances which, like ice, cast iron, and many alloys, contract on fusing and expand during solidification, and this enables sharp castings to be taken in these materials.

162. **Latent Heat of Fusion.** During the process of liquefaction, the application of heat has to be continued, and a large amount is absorbed by the melting body. This heat has no effect upon the *temperature* of the body, which, as we have said, remains constant till the whole is liquid. On this account the heat absorbed during this process was said to become *latent*, and was called the *latent heat of fusion*. It may be thus defined:—

DEF. *The latent heat of fusion of a substance is the number of units of heat absorbed by the unit of mass of the substance in passing from the solid to the liquid state without change of temperature.*

In Chapter III. we saw that a pound of ice in melting at  $0^{\circ}\text{C}$ . would cool a pound of water from  $79^{\circ}\text{C}$ . to  $0^{\circ}\text{C}$ .

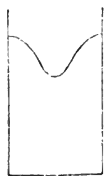


The latent heat of fusion of ice, or, as it is more generally called, the latent heat of water, is therefore about 79, or more accurately, 79·25.

163. The phrase "latent heat" is not well chosen, because the heat absorbed during liquefaction does not remain within the substance as hidden heat, or heat insensible to the thermometer, but actually ceases to be heat, being used up in doing *internal work*; just as when a steam crane is employed in lifting building stones, part of the heat generated in the furnace ceases to be heat, being used up in doing work, and having its representative in so many tons of material lifted so many feet against the attraction of the earth. It was, however, Black's doctrine of *latent* heat, as originally enunciated, which led James Watt (see Art. 205) to see that in Newcomen's engine the necessity of raising the temperature of the steam cylinder and piston to that of steam during each stroke, after cold water had been injected into the cylinder to condense the steam employed in the previous stroke, led to an enormous consumption of steam beyond that required to fill the cylinder and do the work of the engine; and in the model on which Watt experimented this effect was more marked than in a large engine, in which the capacity of the cylinder bore a greater ratio to the volume of metal employed in its construction. Watt concluded that if the working cylinder could always be kept at or near the temperature of the entering steam, all this waste would be obviated. This necessitated that the steam should be condensed in a separate vessel, which might be kept always at a low temperature by means of cold water, and connected with the working cylinder at the end of each stroke, when it became necessary to remove the steam pressure from under the piston. Thus the separate condenser was invented, by which the efficiency of the engine was raised to such an extent as to make it possible to employ it in factories and mills. In this way the modern steam engine became the product of the intercourse between James Watt, the practical instrument maker, and the Professors of the University of Glasgow.

**164. Change of Volume on Solidification.** There are some substances which, like ice and cast iron, expand on solidification. The density of ice at  $0^{\circ}$  C. is  $\cdot 918$ , that of water at  $4^{\circ}$  C. being taken as the standard of density; but as its temperature is lowered the ice contracts, its coefficient of cubic expansion per  $1^{\circ}$  C. being about  $\cdot 000122$ . Those metals which expand on solidifying are well adapted for making sharp castings, since on solidification they expand and fill all the interstices of the mould. On the other hand, it is impossible to obtain sharp castings with metals like gold, silver, lead, etc. For this reason coins, medals, rifle-bullets, etc., are stamped. When a leaden bullet is cast there is always an empty space inside it, due to the contraction of the lead, and the resultant resistance of the air then not generally acting through the centre of gravity of the bullet, its direction of motion is thereby changed. It is

FIG. 27.



to obviate such action that a motion of rotation is given to bullets and cannon-shot by rifling the guns; but in addition to this rifle bullets are always punched out of cold lead instead of being cast. Paraffin contracts very much on solidifying, and if a cylindrical vessel be filled with melted paraffin and allowed to cool, the upper surface of the paraffin will be depressed in the centre, as shown in Fig. 27, while castings made in closed moulds will generally be found interspersed with cavities.

**165. Regelation.** It was first shown from theoretical considerations by Prof. James Thomson, that if a body expand on solidification an increase of pressure will lower the melting point, but if it contract on solidifying, like wax and paraffin, the melting point will be raised by pressure. Sir W. Thomson subsequently found that in the case of ice the melting point is lowered by about  $\frac{1}{135}^{\circ}$  C. for each additional pressure of one atmosphere. This dependence of the melting point upon the pressure explains the phenomenon of regelation.

An example of this phenomenon is presented by the following experiment, due to Mr. Bottomley. A block of

ice is supported in any convenient manner at each end, and a wire loop carrying a weight is suspended from the block, the temperature of the air being above the freezing point. The ice immediately underneath the wire being then exposed to a pressure greater than that of the atmosphere, while its temperature is not sensibly below the freezing point, melts, and the liquid so formed escaping to the upper side of the wire, where the pressure is only that of the air, while the temperature of the water is below the freezing point, it again freezes. (That the temperature of the water must be below the freezing point is obvious, for in melting it must absorb the necessary latent heat from the surrounding ice, cooling it below the freezing point; and it is this cooling which prevents the wire from cutting through the ice instantly, for the cooled ice is able to support the pressure of the wire, and the rate at which the cutting goes on is determined by the rate at which the heat absorbed from the ice below the wire can be again communicated to it as the water freezes above the wire.) Since, with the exception of a little which escapes at the edges, all the water again freezes above the wire, as the block is cut through it becomes re-united, and when the wire has passed completely through it, the only trace left of its passage is a shallow groove surrounding the block, due to the escape of water from the surface, and a striated appearance within the ice along the plane of section.

If a number of wires of the same diameter but different material be hung over the same rectangular block of ice, and loaded with equal weights, it will be found that the wires will pass through the ice in the order of their thermal conductivities. Thus, if wires of copper, brass, and iron be employed, they will cut through the block in the order in which they have just been mentioned.

166. If pieces of ice be compressed by hydraulic pressure in a boxwood mould, partial liquefaction takes place on account of the pressure, and some of the water escapes, carrying with it the latent heat of fusion, or more properly speaking, the energy equivalent to that latent heat. On relieving the pressure the water filling the interstices

between the pieces of ice again freezes, and a solid and transparent block of ice of the form of the mould is formed. Blocks of ice floating in warm water will freeze together at points where they strike each other, for under these circumstances there will be a film of water extending over the surface of the ice, and after the impact, as the blocks tend to recede from one another, the surface tension of the water, or capillarity, will tend to hold them together, and the pressure within the film of water between the blocks will be diminished, and the water will freeze, though its temperature may be slightly above the freezing point. The binding of snow when just at the freezing point may be due to the same cause. If the temperature of ice be much below the freezing point very great pressure is required to melt it, and this may account for the difficulty of making snow-balls when the snow is "frozen."

167. If a bar of wood or any other material be bent, one side of it is compressed while the opposite side is extended. If an attempt be made to bend a bar of ice, the temperature of which is the freezing point, wherever there is pressure the ice will be partially melted, the consequent contraction at once relieving the pressure and allowing the ice to be deformed, while the conduction of heat through the block tends to keep the temperature the same throughout, and the water gradually finds its way to parts where the pressure is less. In the case of a bar subjected to bending stress: when the portion which is most compressed begins to melt, the greatest pressure is thrown upon the next layer, causing it to melt and accommodate the stress; and so on through the block. The production of any considerable flexure will of course take a long time, depending as it does to some extent on the passage of heat through the mass; but it is plain that if the temperature be exactly the freezing point for atmospheric pressure, a stress, however small, will continue to produce change of form so long as it acts, and the ice will therefore behave like a very viscous fluid instead of a solid.

What has just been stated will go far to explain the

motion of glaciers; but in their case fracture of the ice no doubt frequently occurs in passing round a bend, but on coming to a bend in the opposite direction, the crevasses thus formed are closed, and the pressure causes the surfaces to melt, while part of the water escapes; and then regelation takes place, completely healing the wound and obliterating its traces.

168. **Supersaturation.** As a general rule, if a liquid be cooled to the melting point, the process of cooling cannot be carried further without solidification taking place, but water may be cooled considerably below the freezing point if it be carefully freed from air by long boiling and the surface be protected by a film of oil. In capillary tubes water has been cooled by Despretz to  $-20^{\circ}\text{C}$ . without solidifying. When water has been cooled in this way, if it be disturbed so that solidification commences, a net-work of ice crystals is suddenly formed throughout the mass, and the temperature rises to  $0^{\circ}\text{C}$ . A similar action takes place in the case of certain saline solutions, such as that of sulphate of soda (Art. 79), which may be cooled without crystallizing, though at the lower temperature they contain much more salt in solution than can be dissolved by water at that temperature. On disturbing such solutions a mass of crystals is quickly formed, extending completely through the liquid, while the temperature rises on account of the production of heat by crystallization (latent heat of solution).

169. **The Laws of Fusion.** The laws of fusion may be thus stated:—

(1) *Each substance begins to melt at a certain temperature, which is constant for the same substance if the pressure be constant, and is called the melting point.*

(2) *The temperature of the solid remains constant from the time when fusion commences until it is complete.*

(3) *If a substance expand on solidifying. e.g., ice, its melting point is lowered by pressure; but if it contract, e.g., wax, its melting point is raised by pressure.*

(4) *The unit mass of each substance in passing from the solid to the liquid condition without change of temperature, absorbs a certain quantity of heat which is constant for*

*the same body melting at the same pressure and temperature, and is called the latent heat of fusion.*

170. Some substances, on being heated to their melting points, do not become sensibly softened, but pass from the solid to the liquid state without passing through any intermediate viscous stages. Others, like sealing wax, become gradually softer as their temperature is increased, and it is impossible to determine when they pass from the solid to the liquid condition. Sealing wax, in fact, behaves more like a very viscous fluid than a solid even at ordinary temperatures. Among substances which behave in this manner may be mentioned wrought-iron, steel, and platinum. These substances become soft and viscous at temperatures far below those at which they become fluid, and it is in virtue of this property that they are capable of being welded. The range of temperature over which steel is capable of being welded is much less than the corresponding range for wrought-iron, and diminishes as the steel is more highly carbonised. Steel suitable for cutting tools burns vigorously in air, and becomes seriously injured at a temperature very slightly above the lowest temperature at which it can be welded. Hence very great care is needed on the part of the smith, and welds in steel are always regarded with suspicion when great strength is required.

171. Plumbers' solder, consisting of two parts of lead and one of tin, retains a pasty consistency for a wide range of temperature below that at which it is fluid. Over this range the solder can be easily moulded by pressure such as can be applied by the hand of the workman through the medium of a piece of thick cloth. It is in this way that "wiped joints" are made in plumbers' work.

It frequently happens that the melting point of a mixture is considerably below that of the most readily fusible of its constituents. Combinations of iron and carbon, for example, melt at temperatures far below the melting point of pure iron. A mixture of ice and salt melts at a temperature several degrees below the freezing point. Mixtures of lead and tin in various proportions,

forming solders, melt at temperatures considerably below the melting point of lead ( $327^{\circ}\text{C.}$ ), and some of them at temperatures much below the melting point of tin ( $232^{\circ}\text{C.}$ ). Thus a mixture of three parts of tin and two of lead melts at  $166^{\circ}\text{C.}$ , and can be used as a solder for pure tin. When the amount of tin is increased much beyond 60 per cent., the melting point of the alloy is raised.

**172. Melting Points.** The melting points of several substances are given in the following table:—

TABLE OF MELTING POINTS OF ELEMENTS.

Platinum. . . . .	$2000^{\circ}$	Cadmium . . . . .	$320^{\circ}$
Gold . . . . .	$1200^{\circ}$	Bismuth . . . . .	$265^{\circ}$
Cast iron. . . . .	$1150^{\circ}$ to $1200^{\circ}$	Tin . . . . .	$232^{\circ}$
Copper. . . . .	$1090^{\circ}$	Sulphur . . . . .	$113^{\circ}$
Silver . . . . .	$1000$	Sodium . . . . .	$97^{\circ}$
Aluminium. . . . .	$750$	Potassium. . . . .	$58^{\circ}$
Antimony . . . . .	$432$	Phosphorus . . . . .	$43^{\circ}$
Zinc . . . . .	$360^{\circ}$	Bromine . . . . .	$-21^{\circ}$
Lead . . . . .	$327^{\circ}$	Mercury . . . . .	$-39^{\circ}$

MELTING POINTS OF SOLDERS.

Lead, 3 parts, Tin, 1 part (Coarse solder) . . . . .	$250^{\circ}\text{C.}$
" 2 " " 1 " (Plumbers' solder) . . . . .	$227^{\circ}\text{C.}$
" 1 part " 1 " (Fine solder) . . . . .	$186^{\circ}\text{C.}$
" 2 parts " 3 parts . . . . .	$166^{\circ}\text{C.}$
" 1 part " 2 " . . . . .	$171^{\circ}\text{C.}$
" 1 " " 3 " . . . . .	$177^{\circ}\text{C.}$
" 3 parts " 3 " Bismuth, 1 part . . . . .	$157^{\circ}\text{C.}$
" 5 " " 3 " " 8 parts . . . . .	$100^{\circ}\text{C.}$
" 3 " " 2 " " 5 " . . . . .	$93^{\circ}\text{C.}$
" 6 " Cadmium, 1 part " 7 " . . . . .	$82^{\circ}\text{C.}$

**173. Vapours and Gases.** Most liquids and some solids when exposed to the air pass gradually into the gaseous state. The process by which liquids and solids assume the gaseous condition at their free surfaces is called *evaporation*, and the gas so formed is frequently called a *vapour*. It is not easy to distinguish satisfactorily between vapours and gases, but the term vapour is generally employed for a substance in the gaseous condition which can exist as a liquid or solid at ordinary pressures and temperatures. This is the popular use of the terms, but the true distinction between a vapour and

a gas lies in the fact that a vapour can be condensed to a liquid by the application of pressure alone, while a gas cannot be condensed to a liquid by the application of any pressure unless its temperature is reduced. There is therefore a particular temperature above which a substance must be a gas, but below which it may be a vapour or a liquid, according to the pressure. This temperature is called the *critical point* for the particular substance, and will be referred to again shortly. The ordinary temperature of the air is far above the critical points of the more perfect gases. In the case of a liquid the converse of evaporation, or the passage of a substance from the gaseous to the liquid condition, is called *condensation*. When a gas passes directly into the condition of a solid, it is said to *sublime*, and the process is called *sublimation*. A solid or liquid which passes readily into the gaseous condition is said to be *volatile*.

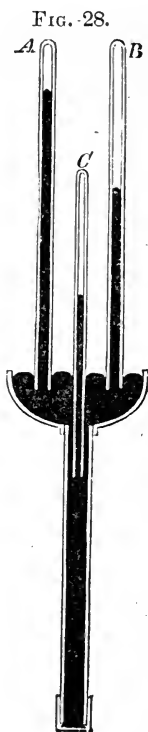
**174. Evaporation.** The evaporation of water is a phenomenon which is continually coming under our notice, and the production of liquid water by the condensation of aqueous vapour is scarcely less familiar to us. Sometimes during a long frost a quantity of snow is observed slowly to vanish though the temperature may never rise to the freezing point, so that the production of liquid water is impossible: the snow in fact presents us with a case of the direct evaporation of a solid. When hoar-frost is produced by the condensation of aqueous vapour upon surfaces cooled below the freezing point, we have an example of sublimation. Solid carbonic anhydride, camphor, iodine, and arsenic readily evaporate without liquefaction and sublime upon colder surfaces. Flowers of sulphur are formed by the condensation of sulphur vapour in a large cold chamber. Arsenic, carbonic anhydride, and some other substances can be obtained in the liquid condition only when subjected to pressures considerably above that of the atmosphere.

**175. Dalton's Law.** If a quantity of water or other volatile liquid be placed in a closed vessel, evaporation will take place until the pressure due to the vapour alone attains a certain limit, which depends only upon the



temperature. The quantity of vapour in the enclosure will then cease to increase, though we do not say that evaporation will cease, but rather that condensation will take place at the same rate as evaporation, and hence the amount of vapour will remain constant. The quantity of vapour which could exist under given conditions in a limited space was for a long while a matter of inquiry, and Dalton was the first clearly to enunciate and verify experimentally the laws of evaporation. Dalton found that *the greatest amount of vapour which can exist in a given space depends only upon the temperature, and is independent of the presence of any other vapour or gas which has no chemical affinity for it.* According to Regnault this law is not strictly true, but it is sufficiently accurate for all the requirements of meteorology and volumetric analysis. It is sometimes expressed by saying that *gases and vapours act towards one another as vacua.* Herwig, on the other hand, concludes that Dalton's law is strictly true, and that Regnault's results are to be explained by the nature of his experiments.

176. Dalton's law may be easily verified experimentally by a method differing but slightly from that adopted by Dalton. The apparatus is represented in Fig. 28, and consists of three barometer tubes with suitable supports, a mercury-bath about 3 feet deep nearly filled with mercury, and a scale or cathetometer for measuring the height of the mercury in the tubes. A yard of inch wrought-iron gas-tubing with a cap screwed to the lower end, and a cast-iron or wrought-iron basin screwed to the upper end, constitutes a convenient mercury-bath. One of the tubes, *A*, is filled with mercury and then placed vertically with its lower end beneath the surface of the mercury in the bath. This serves as a standard barometer for purposes of comparison. The



tube *B* is filled and supported in the same manner, while *C* is only partially filled with mercury, so that when inserted in the bath such a quantity of (dry) air occupies the tube as to cause the mercury to stand at about one-half the height of the barometer. The position of the surface of the mercury in *C* should be carefully marked on the side of the tube. Now let such a quantity of ether be introduced by means of a pipe into the tubes *B* and *C*, that a little of the liquid remains on the surface of the mercury. If the temperature be  $15^{\circ}\text{C.}$ , the column of mercury will be depressed below that in the standard barometer by about 354 millimetres; and if the tube *B* be slowly lowered into the mercury-bath, the mercury will rise in the tube, the ether vapour being condensed, but the surface of the mercury will always remain 354 millimetres below that in the standard barometer till the whole of the vapour is condensed and the tube completely filled with mercury and liquid ether. On again slowly raising the tube, the ether will evaporate, the surface of the mercury remaining at the same height as before, below that in the standard tube *A*. If the tube be lowered *rapidly*, the condensation of the ether vapour will cause a rise in the temperature, and the pressure of the vapour will be increased. If the tube be rapidly raised, the pressure of the vapour will be diminished through the cold caused by evaporation.

On the introduction of the ether into the tube *C*, the mercury will be seen to fall, but not so far as in *B*, because as the mercury falls the air in the tube expands, and its pressure is consequently diminished. Now let the tube *C* be lowered in the bath until the mercury rises to the *point marked on the tube*, and the air consequently occupies the same volume and exerts the same pressure as before. It will then be found that the tube has been lowered through 354 millimetres, or the mercury in the tube stands at a lower level than before the introduction of the ether by 354 millimetres; and since the pressure of the air is unchanged, it follows that the ether vapour must exert a pressure equivalent to 354 millimetres of mercury, and therefore exerts the same pressure in the presence of the air as in the tube *B* which contains no air.

177. If a little alcohol be introduced into the tube *B*, it will immediately absorb some of the ether vapour and cause the mercury to rise on account of the great affinity which alcohol possesses for ether. Bisulphide of carbon and many other liquids act similarly, showing that the pressure of the vapour produced by a mixture of two liquids which dissolve one another may be less than that of their more volatile component.

For rough purposes it is convenient to close the upper ends of the barometer tubes by two glass taps a small distance apart, or to attach to them a short piece of india-rubber tube with two pinch taps upon it, a small funnel being placed at the top. This arrangement enables the tubes to be filled with mercury by pressing them down into the bath till the mercury rises above the upper tap, the taps being open, and then raising them with the taps closed. By pouring the ether into the funnel, and then opening first the upper tap and subsequently, after closing this, the lower tap, a small quantity of ether can be passed into the tube.

178. Dalton, Gay-Lussac, and afterwards Regnault, measured the pressure of aqueous vapour in the presence of water at temperatures below  $100^{\circ}$  C. by the depression of the mercury in barometer tubes, as above described, the experimental tube being surrounded by a bath of hot water, by which its temperature could be adjusted; while the mercury was observed through the glass front of the bath, and corrections made for the expansion of the mercury itself. For very low temperatures, Gay-Lussac bent the top of his tube so as to incline it downwards, and dipped the end into a freezing mixture, since the pressure of the vapour must always be that due to the temperature of the *coldest part of the tube* to which it has access. Regnault followed the same method, and some of his results are given in the Table in Art. 202.

179. **Distillation.** The fact that the pressure of aqueous vapour in any space must be that due to the coldest part of the space to which the vapour has access, explains the process of *distillation*. If two vessels *A* and *B*, of which *A* contains a volatile liquid, be connected by

a tube, and *A* be maintained at a temperature above that of *B*, the vapour in *B* will be condensed until its pressure is not greater than that due to the temperature of *B*, while evaporation will go on in *A* so long as the pressure of the vapour there is less than that due to its temperature; moreover, vapour will pass from *A* to *B* if the pressure of the vapour in *A* be greater than in *B* (though if the vessels contain air or other gas, the diffusion of the vapour will be a slow process). Hence, evaporation will go on in *A*, and condensation in *B*, so long as the temperature of *A* is higher than that of *B*.

The same fact explains the efficiency of the "separate condenser" referred to in Art. 163.

180. **The Cryophorus.** Wollaston's cryophorus consists of two glass bulbs connected by a tube, and containing sufficient water to about half fill one bulb. The water is boiled to expel all the air before the tube is finally sealed, and when this has been done, the sealing is effected with a blow-pipe flame. If the water be turned into the bulb *A*, and *B* be placed in a freezing mixture of ice and salt, condensation will take place so rapidly in *B* and evaporation in *A* that, on account of the heat absorbed in the latter process, the water in *A* will become completely frozen.

181. **Boiling.** As its temperature rises, the pressure which a vapour in contact with its liquid is capable of exerting goes on increasing, while the density of the vapour itself increases in consequence of increased evaporation. If a liquid be exposed to the open air while its temperature is gradually raised, the pressure which the vapour can exert will at length exceed that of the air; and then, if a bubble of pure vapour be formed within the liquid itself, it will rise to the free surface and escape, since the pressure to which it is exposed is insufficient to condense it. When bubbles of *pure* vapour are formed within the liquid itself, the liquid is said to *boil*. This may happen as soon as the temperature has risen so far that the pressure which the vapour can exert exceeds atmospheric pressure.

DEF. *The temperature at which the pressure which can be exerted by a vapour in the presence of its liquid is equal to that of the standard atmosphere is called the boiling point of the liquid.*

182. The difference, then, between evaporation and ebullition, or boiling, consists in this: that in evaporation vapour is formed only *at the free surface* of the liquid, while in ebullition bubbles of pure vapour are formed *within the liquid itself*.

183. When a kettle is being heated, the water next the bottom first reaches the boiling point, and bubbles of steam are formed there. These rising through the liquid reach a stratum where the temperature is insufficient to allow the steam to exist at the temperature to which it is exposed, and the bubbles contract with a sharp sound. These sounds, happening with sufficient frequency, link themselves into a musical note, producing the *singing* of the kettle, which gets feebler as the bubbles contract less sharply, and is presently exchanged for the smooth rolling sound of ebullition as the bubbles at length escape from the free surface.

184. When a liquid boils, so long as the pressure to which it is exposed remains the same, the temperature of the vapour immediately above the surface remains constant, though the temperature of the liquid itself may be somewhat different from that of the vapour. The presence of salts in water raises the temperature of the water; and this temperature, moreover, depends on the nature of the containing vessel. Hence, in determining the boiling points on thermometers, they are immersed in the steam and not in the water itself (Art. 13).

185. In concentrating solutions of caustic soda, cast-iron pans are employed, and towards the end of the operation, when the contents of the pan approximates to pure sodic hydrate, the temperature reaches a red heat. Very concentrated solutions have, when cooled somewhat below the highest temperature at which they have been concentrated, a great affinity for water, and if steam be passed into them the combination of the soda and steam develops a very great amount of heat. This property has recently

been employed for developing steam in locomotive engines, especially for town tramways. The boiler, which is filled with distilled water to prevent incrustation, is of the tubular type, and is placed inside an iron vessel into which a very concentrated solution of caustic soda is introduced, the temperature of the solution being sufficient to produce steam at the requisite pressure, but considerably below the highest temperature at which the solution has been concentrated. The high-pressure steam produced in the tubular boiler is employed in the cylinders, and the exhaust steam is passed into the caustic soda solution, there to be condensed through the affinity of the solution for water, and the heat produced by this condensation and combination keeps up the temperature and continues the production of steam. The source of heat in the engine, therefore, is not simply the heat of the solution of soda as introduced, but that developed by the combination of the soda solution with the water originally in the boiler, and the engine will continue to work until the solution has become so dilute as no longer to be able to maintain the necessary temperature by condensing the exhaust steam. Some engines can draw a load for 50 miles with a single charge of soda. The same soda solution appears to act both as furnace and condenser, but in reality the source of heat is the furnace by which the solution was concentrated, and in comparing the engine with Carnot's, the temperature of the source of heat should be taken as the highest temperature to which the solution is raised in concentrating it.

186. **Superheating Water.** It was stated in Article 181 that if a bubble of vapour be formed within a mass of liquid whose temperature is above the boiling point, and the liquid be subjected to atmospheric pressure, the bubble will not be condensed, but it does not follow that such bubbles will be formed immediately after the boiling point is reached. If water be very carefully freed from air by boiling, it is possible to heat it considerably above the boiling point without the formation of bubbles of steam, on account of the resistance offered by the water to any breach in the continuity of its mass, such as would

be produced by the formation of steam bubbles within it. The boiling by bumping which often takes place with water, and especially with strong alkaline solutions, is due to this superheating of the water, and the consequent generation of a large quantity of steam suddenly. The introduction of a piece of metal which can decompose water, invariably sets free steam with explosive violence, and the temperature falls nearly to the boiling point. The presence of oil on the surface of the water favours the superheating; and Dufour, by suspending drops of water in a mixture of linseed oil and oil of cloves, succeeded in raising their temperature to about  $180^{\circ}\text{C}$ . without ebullition, though the pressure of steam in the presence of water at that temperature is equal to nearly ten times the standard atmosphere. It is so difficult to remove the last trace of dissolved air and other gases from water, that it has been said that the phenomenon of pure water boiling has not yet been witnessed.

187. **Hypsometry.** Since the temperature of steam above the surface of boiling water depends only on the pressure, it follows that if we can determine the temperature of steam above the surface of boiling water at any place, we may find, by reference to tables constructed for the purpose, the barometric pressure to which it is subjected. The *hypsometer*, described in Art. 12, enables us to do this, and may therefore be employed instead of a barometer for the determination of the heights of mountains. It is from its employment for this purpose that the instrument derives its name.

188. **Boiling in a Closed Vessel.** We have said that the temperature at which a liquid boils depends upon the pressure to which it is exposed, and have defined the boiling point of a liquid as that temperature at which the pressure of its vapour is equal to the atmospheric pressure. If the temperature of the liquid be higher than this, vapour may be generated within its mass, and the liquid will then boil. If the vessel containing the liquid be open, the boiling will continue as long as there is any liquid present at a temperature above that at which it can boil under the atmospheric pressure;

but if the liquid be in a closed vessel, its own vapour will increase the pressure upon it, and presently the boiling will cease, provided the temperature be kept constant.

If there be no air, or gas other than the vapour of the liquid, above its surface, the whole pressure on the free surface of the liquid is due to the pressure of its vapour, and the amount of vapour in the unit of volume will therefore be that which is capable of exerting this pressure at the temperature of the enclosure. Since the pressure under which liquids boil increases very rapidly with the temperature, much more so than the pressure of a given mass of vapour at constant volume, it follows that the density of the vapour above the surface of the liquid will increase rapidly with the temperature.

189. If a quantity of warm water at a temperature considerably below  $100^{\circ}$  C. be placed under the receiver of an air-pump, and the air rapidly removed, the water will boil violently, showing that the temperature at which water boils is lowered by diminution of pressure.

The same may also be shown in a striking manner by boiling a quantity of water in a glass flask until the air above it has been expelled by the steam, corking the neck of the flask, and pouring a stream of cold water over it. The water in the flask will then be seen to boil vigorously. In this case, the steam in the upper part of the flask is cooled much more than the mass of water below it. Its temperature is therefore reduced below the boiling point corresponding to its pressure, and part of the steam is consequently condensed. On account of this, the pressure on the surface of the water is diminished, but the temperature of the water is, for some time, only slightly lowered by the stream outside; hence its temperature is sensibly above the boiling point corresponding to the pressure on its surface. The water therefore boils violently, the steam produced being in its turn condensed; and this process can be carried on for a considerable time.

190. Now suppose that a quantity of liquid is placed in a closed vessel, and that there is *dry air* above its surface. Suppose also that the temperature of the water is



sensibly above the boiling point corresponding to the pressure of the air above it, and is kept constant. The liquid will then boil violently, until the pressure on its surface is equal to the maximum pressure of vapour at the temperature which the liquid possesses. When this stage is reached, the pressure on the liquid is the sum of the pressures exerted by the air and by its own vapour, and the greater the pressure of the air the less will be the amount of vapour per unit volume *when the liquid just ceases to boil*.

It is found, however, by experiment, that the formation of vapour in a closed vessel, when air, or any gas other than the vapour, is present, does not stop when the liquid ceases to boil, but *goes on until the pressure exerted by the vapour alone is the maximum pressure which it can exert at the temperature of the liquid*. During this process no bubbles of vapour are formed in the liquid, as is the case during boiling, but the vapour appears to be quietly formed at the surface; that is, by *evaporation*.

191. Thus, in accordance with Dalton's law, a liquid will go on evaporating till the pressure of its vapour alone reaches a certain value *depending only on the temperature*, and independent of the presence of any amount of other vapours, or gases, in the same space. If a quantity of water be placed in a closed vessel with dry air above it, though the pressure of the air may be far too great to allow of the water boiling, nevertheless vapour will be formed till the amount above the water is the same as if no air had been present, the only difference being that, in the case we have supposed, the whole of the vapour is produced by slow *evaporation*, while if the water had been placed in vacuo its formation would have been much more rapid and accompanied with *ebullition*. We also see that the pressure on the water will finally exceed that exerted by the air, however great this may be, by a quantity depending only on the temperature as above stated. This is sometimes expressed by saying that different gases and vapours act towards one another as vacua.

192. **Rate of Evaporation.** The rate at which evaporation appears to take place depends upon the tem-

perature of the liquid and the rate at which vapour can escape from the neighbourhood of the surface; hence, movements of the air increase the apparent rate of evaporation. According to Dalton, the rate at which water evaporates in calm air at ordinary pressure is proportional to the difference between the maximum pressure of vapour at the temperature of the water and that of the vapour existing in the air.

The apparent rate of evaporation is really the difference between the rate at which particles leave the liquid and become gas, and that at which particles of gas enter, and remain within, the liquid; that is, the difference between the rates at which evaporation and condensation take place.

**193. Pressure of Vapour Dependent on Temperature.** If a quantity of liquid be placed in a closed vessel, and gradually heated above the boiling point, the pressure of the vapour will increase and exceed the atmospheric pressure. The pressure may be measured by causing the vessel to communicate with any convenient pressure-gauge, as, for example, an ordinary manometer or U-tube containing mercury, one of whose arms is open to the air, while the other communicates with the vessel. The difference in the level of the mercury in the two tubes indicates the excess of the pressure of the vapour over that of the air. If the pressures are very great, a continuous syphon-gauge may be employed, consisting of a number of U-tubes connected together, and half filled with mercury, while the spaces between the mercury in consecutive tubes are filled with water or other nearly incompressible fluid. The sum of the differences in level of the surfaces of the mercury in the arms of each tube, added to the height of the barometer, and diminished by the column of mercury which is balanced by the water or other liquid, indicates the pressure of the vapour.

**194.** Since it is generally impossible to raise the temperature of water above that at which it boils under the pressure to which it is exposed, in order to raise it to high temperatures it must be confined in a closed vessel. Papin's digester is a strong vessel (furnished with a safety-

valve), in which water can be raised to a temperature considerably above  $100^{\circ}\text{C}.$ , for the purpose of dissolving gelatine from bones, etc. On high mountains water boils at a temperature so low as seriously to interfere with culinary operations. Thus, on the top of Mont Blanc the temperature of water boiling in the air is not much above  $82^{\circ}\text{C}.$

Sometimes it is of importance to cause liquids to boil at temperatures below their ordinary boiling points, and for this purpose it is necessary to reduce the pressure to which they are exposed below the ordinary atmospheric pressure. The introduction of the vacuum pan in the refining of sugar, enabling the syrup to boil at about  $66^{\circ}\text{C}.$ , allowed the process to be carried on without the production of the large amount of uncrystallizable sugar which always took place when the boiling was conducted at a higher temperature.

**195. The Dew-Point.** DEF. *When the amount of any vapour in a space is the greatest that can exist in it without increase of temperature, the space is said to be saturated with the vapour.*

DEF. *The temperature at which the air is saturated with the aqueous vapour it contains is called its dew-point.*

If the temperature of a quantity of air containing an amount of aqueous vapour insufficient to saturate it be lowered, the air will at length become saturated, and the corresponding temperature is the dew-point. On still further lowering the temperature, condensation of part of the vapour takes place and the air remains saturated.

**196.** From what has been said it will be seen that air in the presence of water must contain a quantity of aqueous vapour; and thus all the air around us contains more or less vapour, some of which will be condensed if the air be sufficiently cooled, condensation commencing when the dew-point is reached.

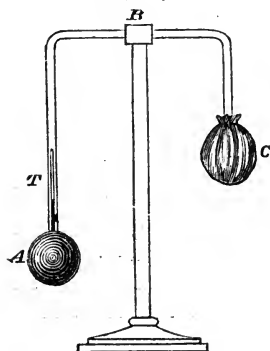
If we have a table giving us the pressure of aqueous vapour for different temperatures in a space saturated with it, or what is the same thing, the boiling point of water corresponding to different pressures, the determina-

tion of the amount of aqueous vapour in any volume of air may be reduced to a determination of the dew-point. The determination of this point is therefore a matter of considerable importance, and several instruments have been constructed for the purpose.

197. A simple form of dew-point instrument, introduced by Mr. Dines, consists of a metal box covered with a piece of polished black glass, a thermometer being placed with its bulb in close contact with the glass. A stream of cold water, whose rate can be regulated at pleasure, is made to flow through the box, and when a film of moisture begins to render dull the polished surface of the glass, the stream is stopped and the thermometer read. In a few seconds the cloud will begin to leave the glass, and the thermometer is again observed. The mean of the two temperatures is approximately the dew-point.

198. Daniell's dew-point instrument, frequently called Daniell's hygrometer, is shown in Fig. 29. A bulb *A* made of black glass is connected with the bulb *C* by the tube *ABC*, of clear glass. The bulb *A* contains a quantity of liquid ether, into which dips the bulb of a small thermometer *T*. The whole apparatus is exhausted of air

FIG. 29.



before being sealed, so that the bulbs contain only ether and its vapour. The bulb *C* is covered with muslin or similar material, and when the instrument is to be used, liquid ether is poured over the muslin. This evaporates, and in so doing absorbs the latent heat of evaporation (see Art. 205), cooling the bulb and allowing the ether vapour within it to condense, thus diminishing the pressure within the tube. This causes the ether in *A* to evaporate, with a consequent lowering of the tem-

perature of *A*, and after some time moisture begins to be deposited on the outside of the bulb. The thermometer *T* is then observed, and the muslin on *C* is allowed to

become dry. The temperature of  $A$  will then gradually rise, and when the dew upon it begins to disappear, the temperature of  $T$  is again observed. The mean of the two readings of  $T$  is taken as the dew-point.

199. Regnault's dew-point instrument consists of two test tubes silvered at the bottom. In one of these ether is placed and air blown gently through it until the evaporation has lowered the temperature so that dew begins to be deposited on the silver cap. The second tube is merely for purposes of comparison, in order the more readily to detect the first appearance of dew on the former tube. A thermometer is placed with its bulb in the ether.

200. Mason's dry and wet bulb thermometers, sometimes called August's psychrometer, are frequently used for determining the amount of moisture in the air. They consist of two thermometers mounted side by side, the bulb of one being exposed while that of the other is covered with lamp-cotton, or muslin, kept wet by dipping one end into a vessel of water, when the water rises in the cotton, as spirit in the wick of a lamp, and the bulb of the thermometer is thus kept wet. Now, if the air be far from saturated with moisture, the water will evaporate rapidly from the cotton; and in doing so will cool the bulb of the thermometer below the temperature of the air, the amount of such cooling being less the more nearly the air is saturated. Empirical formulæ, based on the assumption that evaporation continues until the air is saturated at the temperature to which it is reduced, have been employed for the determination of the dew-point from the temperatures indicated by the thermometers, and if the current of air to which the instrument is exposed be gentle the method admits of considerable accuracy.

201. The determination of the dew-point, or of the amount of moisture in the air, is called hygrometry, and instruments employed for this purpose are called hygrometers, or dew-point instruments, as the case may be.

It should be noticed that in all practical determinations of the dew-point the air in the neighbourhood of the dew-

point instrument is cooled at *constant pressure*. Its volume, therefore, diminishes during the cooling, and the volume of the aqueous vapour is correspondingly diminished. Since both air and vapour obey the gaseous laws, it follows that, since the sum of their pressures remains constant, the pressure of the vapour alone will remain constant up to the point when it begins to condense,—that is, the dew-point. The maximum pressure of aqueous vapour at the dew-point as ordinarily determined is therefore the same as the actual pressure of the vapour in the air, and no correction for temperature is required.

**202. Pressure of Aqueous Vapour at Different Temperatures.** When the temperature of a quantity of vapour is above the dew-point, so that the space is not saturated with it, the vapour is sometimes said to be *super-heated*. Regnault has shown that aqueous vapour at temperatures between  $0^{\circ}$  C. and  $25^{\circ}$  C., obeys the gaseous laws with considerable accuracy, provided it is not on the point of condensing. Hence, if we know the dew-point and the temperature of the air, and are moreover furnished with a table showing the maximum pressure of aqueous vapour at different temperatures, we can determine the amount of aqueous vapour existing in a cubic foot of air. Balfour Stewart has proposed to measure the degree of humidity of the air by the ratio of the amount of vapour actually existing in it to that which would be required to *saturate* it at the temperature it possesses. Tables have been constructed by Regnault giving the maximum pressure of aqueous vapour for every tenth of a degree Centigrade near ordinary temperatures, and then for every degree up to  $230^{\circ}$  C. A few of his results are given in the following table, the pressures being expressed in millimetres of mercury at  $0^{\circ}$  C. in the latitude of Paris, and 60 metres above the sea level.

It may be convenient to remember that a cubic yard of steam at  $100^{\circ}$  C. and at standard pressure contains almost exactly one pound. A cubic yard of water at  $4^{\circ}$  C. is very nearly 15 cwt., or 1680 lbs.

TABLE OF PRESSURE OF AQUEOUS VAPOUR.

Temperature.	Pressure.	Temperature.	Pressure.
	Millimetres.		Millimetres.
—32° C.	0·320	90° C.	525·450
—20	0·927	100	760·000
—10	2·093	110	1075·37
— 5	3·113	120	1491·28
0	4·600	130	2030·28
5	6·534	140	2717·63
10	9·165	150	3581·23
15	12·699	160	4651·62
20	17·391	170	5961·66
30	31·548	180	7546·39
40	54·906	190	9442·70
50	91·982	200	11688·96
60	148·791	210	14324·80
70	233·093	220	17390·36
80	354·643	230	20926·40

203. As an example illustrating the subject of hygrometry, we may take the following:—

*Find the weight of dry air in a cubic foot of air containing aqueous vapour at 15° C. under a pressure of 30 inches of mercury, when the dew-point is 10° C., it being given that a cubic foot of dry air at 0° C. and under the above pressure weighs 554 grains, and that the pressure of aqueous vapour in a space saturated with it at 10° C. is equal to the pressure of ·36 in. of mercury.*

The amount of aqueous vapour in the cubic foot is such that at 10° C. it would exert a pressure equal to that of ·36 in. of mercury. The pressure exerted by the dry air occupying the same volume is therefore equal to that of 30 — ·36 in. of mercury, or 29·64 in. of mercury. Hence, the volume which the dry air would occupy under a pressure of 30 inches of mercury at 15° C. is  $\frac{29·64}{30}$  cub. ft., and the volume it would occupy at 0° C. under the same pressure is  $\frac{29·64}{30} \times \frac{273}{288}$  cub. ft., and its weight is therefore  $554 \times \frac{29·64}{30} \times \frac{273}{288}$  grains,

= 521·12 grains, nearly.

**204. Kinetic Theory of Evaporation.** The fact that the amount of aqueous vapour capable of existing in a cubic foot, or other definite amount, of space above the surface of water is independent of the presence of any other vapours or gases, is explained by the kinetic theory in the following way.

The theory supposes that liquid water consists of a number of particles moving with different velocities in all directions, but so acting on one another as to be continually interfering with each other's motions. It also supposes that the vapour consists of a number of particles also moving with various velocities, but not interfering with one another until they come very close together, when repulsion takes place between them. The velocities both of the particles of gas, or vapour, and of those of water, are increased with increase of temperature. The theory also supposes that there is a force acting for a very small distance on each side of the bounding surface of the liquid and gas, tending to prevent the escape of particles from the liquid. If a particle of liquid is moving towards the surface with sufficient velocity, it will have sufficient energy to carry it right through this field of force, and will then be free in space, and, in fact, will be a particle of gas. Now we have said that increase of temperature increases the velocity of the particles of liquid, and it will therefore increase the number which will escape from any surface of the liquid in a second. This number will therefore depend upon the temperature of the liquid, and be also proportional to its surface. Now, of the particles of gas outside, some will be moving towards the liquid and striking it will enter the liquid, where they will be, as it were, taken prisoners by the other particles, and ceasing to be particles of gas will become part of the liquid. Also, the number of those which enter the liquid per second will be proportional to the extent of the surface of the liquid and *to the number of particles of gas in the unit of volume above it.* It will also depend on the temperature of the gas. Now, *when the number of particles of liquid which escape from each unit of area of its surface is equal to the number of particles of the gas which enter it through the same area, the number of particles of gas above the liquid*



*will remain unchanged*; in fact, the space will then be saturated with the vapour, and it may be shown that this final state of things, or balance of exchanges, will not be affected by the presence of foreign particles with the vapour. This explains Dalton's laws, and also shows that the apparent rate of evaporation must depend upon the rate at which the vapour can get away from the liquid by *diffusion* through the air.

**205. Latent Heat of Evaporation.** When a liquid begins to boil, although heat is continually communicated to it and the temperature of the vapour is not above that of the liquid, yet as long as any of the liquid remains its temperature will continue unchanged, provided that its composition be not altered. (This provision excludes unsaturated solutions which become more concentrated on boiling, and mixtures of liquids which undergo fractional distillation.) The heat which enters the liquid during the process of boiling, as it does not raise the temperature, was supposed by Dr. Black to become latent within the vapour, and this hypothesis seemed the more probable since this heat again becomes sensible when the vapour is condensed. We now know that the so-called latent heat really ceases to be heat, being entirely used up in doing work and causing the water to become vapour in opposition to molecular forces and to the pressure of the air. When the vapour is condensed a corresponding amount of work is done upon the particles, and the same amount of heat is again produced.

**DEF.** *The number of units of heat required to convert the unit of mass of a liquid or solid into vapour without change of temperature is called the latent heat of vaporisation, or, more frequently, the latent heat of the vapour.*

**206. Total Heat of Steam.** Soon after Dr. Black enunciated his theory of latent heat, James Watt found that the latent heat of vaporisation of water diminishes as the temperature rises. The amount of heat required to raise the unit mass of water from  $0^{\circ}\text{C.}$  to any particular temperature, and then to convert it into saturated steam, Regnault called the *total heat* of steam at that tempera-

ture. The phrase is sometimes convenient though very unjustifiable. Watt concluded that the total heat of steam is independent of its temperature, which would make the latent heat diminish by unity for every increase of one degree in temperature. Had this been the case the heat required to produce a pound of steam would have been the same at all temperatures and pressures; but the latent heat does not diminish so rapidly as this, and the total heat therefore increases with the temperature. Regnault found that the total heat of steam at any temperature,  $t^{\circ}\text{C.}$ , may be represented by

$$606.5 + .305t.$$

Hence, to produce from ice-cold water a pound of steam at atmospheric pressure requires 637 units of heat; but to produce a pound of steam at a pressure of 10 atmospheres and temperature of  $181^{\circ}\text{C.}$  requires only 661.7 units of heat.

207. Assuming that the specific heat of water is always unity (which is not quite true), it follows that the latent heat of steam at  $t^{\circ}\text{C.}$  must be equal to the total heat diminished by the  $t$  units required to raise the temperature of the water to  $t^{\circ}$ . The latent heat of steam at  $t^{\circ}\text{C.}$  will therefore be represented by

$$606.5 - .695t.$$

Putting  $t$  equal to 0, we have for the latent heat of aqueous vapour at the freezing point, 606.5; while putting  $t$  equal to 100, we see that the latent heat of steam formed at the standard atmospheric pressure is 537.

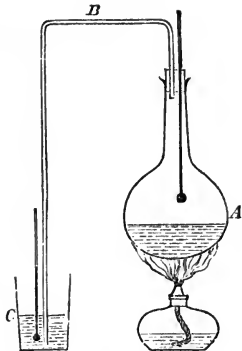
208. It is worthy of remark that (with the exception of certain mixtures of alcohol and water, which have been shown by Dr. Dupré and Mr. Page to have a greater specific heat than pure water) water has a higher specific heat than any other liquid or any solid; the latent heat of fusion of ice is greater than that of any other substance; and the latent heat of steam at the ordinary boiling point is greater than that of any other vapour. The latent heat of alcohol is about 202.4, that of ether about 90.5, and that of bromine about 45.6.

209. Measurement of the Latent Heat of Evapo-

**ration.** The latent heat of evaporation of water, more generally called the *latent heat of steam*, may be measured by means of the apparatus shown in Fig. 30. *A*

FIG. 30.

represents a flask containing boiling water, the steam from which escapes through the tube *B*; *C* is a beaker containing a known weight, say 1000 grains, of distilled water at a known temperature, say  $14^{\circ}\text{C}$ . When the steam is escaping freely from *B*, its end is made to dip nearly to the bottom of the water in the beaker. After a short time the tube *B* is removed, and the temperature of the water in the beaker observed. Suppose it to be  $20^{\circ}\text{C}$ . The beaker is then weighed, and the difference between this and the last weighing gives the amount of steam which has entered it. Suppose this to be 10 grains, and suppose the temperature of the steam, as indicated by the thermometer in *A*, to be  $100^{\circ}\text{C}$ . Then 10 grains of steam in being cooled from  $100^{\circ}\text{C}$ . to  $20^{\circ}\text{C}$ ., have raised the temperature of 1000 grains of water  $6^{\circ}\text{C}$ . Ten grains of steam, therefore, in condensing and cooling from  $100^{\circ}\text{C}$ . to  $20^{\circ}\text{C}$ ., emit as much heat as would raise 6000 grains of water  $1^{\circ}\text{C}$ . But 10 grains of water in cooling from  $100^{\circ}\text{C}$ . to  $20^{\circ}\text{C}$ ., would emit sufficient heat to raise 800 grains of water  $1^{\circ}\text{C}$ . Hence, 10 grains of steam at  $100^{\circ}\text{C}$ ., in condensing to water at  $100^{\circ}\text{C}$ ., emit sufficient heat to raise 5200 grains of water  $1^{\circ}\text{C}$ . The latent heat of steam is therefore determined by this experiment to be 520.



210. The result obtained in the last article is, obviously, too small, because we have neglected the heat taken up by the beaker as well as that lost to surrounding bodies. This source of error may be eliminated by emptying the beaker after the last weighing, placing in it 1000 grains of water at  $14^{\circ}\text{C}$ ., as before, and slowly pouring into this a quantity of water at some known temperature, say  $50^{\circ}\text{C}$ ., keeping the whole well stirred until the temperature of the water in the beaker is  $20^{\circ}\text{C}$ ., the same as after the

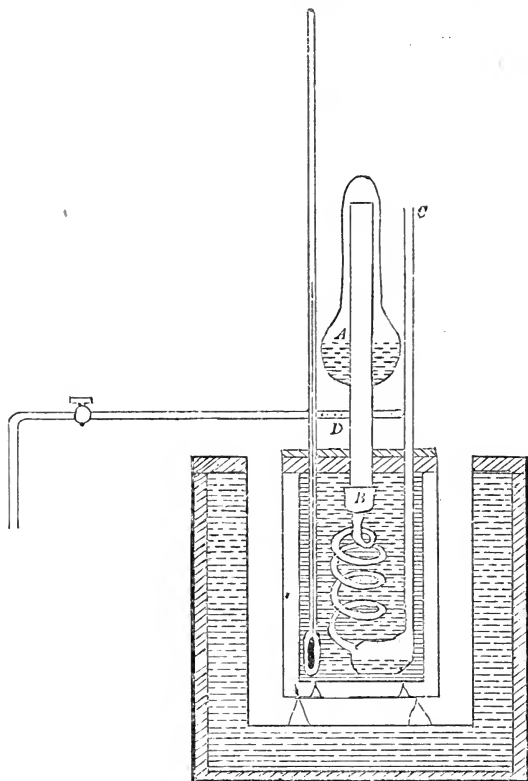
entrance of the steam. If about the same time be occupied in this process as in the admission of the steam, about the same amount of heat will be lost to surrounding bodies, and the same amount will be taken up by the beaker. Suppose the amount of water at  $50^{\circ}\text{C}$ . which has been poured in to be 206 grains. Then, since the 206 grains of water at  $50^{\circ}\text{C}$ . have produced the same effect as was produced previously by the 10 grains of steam, it follows that 10 grains of steam at  $100^{\circ}\text{C}$ ., in becoming water at  $20^{\circ}\text{C}$ ., give out as much heat as 206 grains of water in cooling through  $30^{\circ}\text{C}$ ., or as much as would raise 6180 grains of water  $1^{\circ}\text{C}$ . Whence we get for the latent heat of steam the number 538. Its actual value at  $100^{\circ}\text{C}$ . is about 537.

211. The calorimeter described in Art. 42 may with advantage be substituted for the beaker in this experiment. Care should be taken to protect the upper part of the flask and the tube from loss of heat by a jacket of cotton wool or some similar material, and a screen should be placed between the calorimeter and the source of heat, to keep off radiation.

The cotton wool which surrounds the steam pipe in the apparatus above described does not wholly prevent condensation, and when possible it is better to surround the tube with a steam jacket, though this will require additional precautions to prevent heat being communicated directly to the calorimeter. M. Berthelot overcomes the difficulty by a very neat arrangement. His apparatus, Fig. 30*a*, consists of a flask *A*, into which the liquid whose latent heat of evaporation is to be determined is placed; but the steam pipe, instead of passing out at the top, passes down through the liquid and through the bottom of the vessel into the calorimeter, where it is united with a "worm" at *B*. The extremity of the worm is open to the air at *C*, which secures the pressure in the flask being sensibly equal to that of the atmosphere. The calorimeter is covered with wood and cardboard, to protect it from direct radiation, and the flask is heated by a ring burner *D*, over which is placed wire gauze. The calorimeter is similar in principle to that described in Art. 42,

but is surrounded by a hollow vessel of water to protect the instrument from radiation to or from the objects in the room. A weighed quantity of the liquid is introduced into the flask, and is boiled until the whole has evaporated.

FIG. 30a.



The small amount of heat communicated to the calorimeter by conduction along the steam pipe is determined by observing the thermometer in the calorimeter before the distillation has commenced and after it has been completed.

By making the vessel *A* of sufficient strength, and connecting the tube *C* with a reservoir containing compressed air—the pressure in which is maintained by a pump and registered by a mercury gauge—the latent heat of steam at high temperature and pressure may be determined. By partially exhausting the air in the reservoir by means of an air pump, the latent heat of steam at low temperature and pressure may be measured with the same apparatus.

212. The fact that heat is consumed by water in passing into the condition of vapour prevents the sudden generation of large quantities of steam under ordinary circumstances. If the latent heat of steam were zero, as soon as a quantity of water reached the boiling point the whole would at once be converted into steam, and the heating of water in an ordinary kettle to near the boiling point would be a highly dangerous operation. There are some bodies in which, when they pass into the gaseous condition, such passage is accompanied by a chemical change which allows work to be done by the chemical forces, and heat to be generated instead of being absorbed. Gun-cotton, dynamite, and the like, are examples of such bodies, and they behave very like a substance whose latent heat of vaporisation is negative.

213. **Boiler Explosions.** Suppose a steam boiler to contain 10,000 lbs. of water and steam at a pressure of 150 lbs. per square inch above atmospheric pressure. The temperature will then be nearly  $189^{\circ}\text{C}$ ., and the water will contain rather more than 890,000 units of heat over and above that which it would contain at  $100^{\circ}\text{C}$ . If the boiler were to burst, the pressure would be at once reduced to atmospheric pressure and the temperature to  $100^{\circ}\text{C}$ ., so that steam will be generated until 890,000 units of heat have been absorbed as latent heat. The latent heat of steam at atmospheric pressure is 537; hence about 1657 lbs. of steam will be produced; and since a cubic yard of steam at  $100^{\circ}\text{C}$ . and atmospheric pressure contains 1 lb. very nearly, it follows that 1657 cubic yards, or 44,739 cubic feet, of steam at atmospheric pressure will be generated immediately on the bursting of the boiler.

214. **Condensation.** If a vapour be cooled below the temperature at which its pressure is the greatest which can be exerted at that temperature by the vapour in contact with its liquid, condensation will generally commence; and if the temperature be lowered still further, or attempts be made to increase the pressure of the vapour by diminishing the space occupied by it, the condensation will continue, the space remaining always saturated. There is evidence, however, that vapour cannot condense except there be present some portion of its own liquid or some other suitable surface on which it may be deposited. Thus, air perfectly free from solid or liquid particles may be charged with vapour, and then cooled to a temperature far below the dew-point without any condensation taking place. If there be any motes in the air, such for instance as particles of smoke, etc., these will form foci upon which the vapour will condense, and thus a fog or mist may be created. This tendency of solid particles to condense vapour upon them probably accounts for the prevalence of fogs in large manufacturing towns, especially when situated on the banks of rivers. As a general rule (see *Maxwell's Heat*) evaporation takes place from liquids the more readily the more convex their surfaces. Thus, if a small drop of water with a very convex surface and a large drop with a surface of less curvature be placed in the same receiver, the small drop will evaporate and the vapour will condense upon the larger drop, until the former has disappeared. We may therefore infer that in the case of a drop indefinitely small, if such could exist, evaporation would go on whatever were the pressure of the surrounding vapour; so that the commencement of condensation of a vapour into spherical drops with no nucleus, and therefore with a surface of indefinitely sharp curvature, is an impossibility.

215. Faraday's first apparatus for the liquefaction of gases consisted simply of a strong glass tube slightly bent in the middle, and held so that each half sloped downwards from the middle point. The material from which the gas was to be generated by heat was placed at one end of the tube, which was then hermetically sealed; and on

being warmed the gas was liberated and liquefied at the other (cold) end of the tube by its own pressure. By employing crystals of chlorine hydrate, chlorine gas was liberated and condensed into an oily liquid. With silver ammonium chloride, liquid ammonia was obtained. Faraday afterwards employed force pumps for compressing the gases; forcing them into glass tubes immersed in a mixture of solid carbonic anhydride and ether. With this apparatus he succeeded in liquefying nearly all the gases known to him except oxygen, hydrogen, nitrogen (air), marsh gas, and carbonic oxide. He expressed his opinion that at  $-166^{\circ}\text{F}$ . no amount of pressure would liquefy these gases, and until very recently they were called permanent gases. They obey "the gaseous laws" much more faithfully than do those gases which can be liquefied readily (Art. 119). Since Faraday's time (in 1877) these gases have all been liquefied, and hydrogen solidified, by Cailletet and Pictet. Thilorier's apparatus for the liquefaction of carbonic anhydride consists of two iron bottles connected by a tube. One of these vessels is placed in a freezing mixture of ice and salt, while carbonic anhydride is produced in the other by the action of sulphuric acid on chalk. The gas passes over and liquefies in the cold vessel. If the liquefied carbonic anhydride be allowed to escape into the air through a fine jet, the bottle being inverted for that purpose, the cold produced by the rapid evaporation causes some of the material to freeze, forming carbonic anhydride snow, which may be collected by discharging the liquid into a wooden box perforated so that the gas can escape, when the solid is left in the box. Andrews' apparatus for the condensation of carbonic anhydride will be described in Art. 219.

**216. Carré's Freezing Machine.** A useful illustration of the great amount of heat absorbed during evaporation is afforded by Carré's freezing machine. This consists essentially of two closed iron vessels communicating by a pipe which can be closed by a tap. In one of these vessels, *A*, is placed a concentrated solution of ammonia. The other, *B*, is annular in form, surrounding a hollow space into which the material to be cooled is



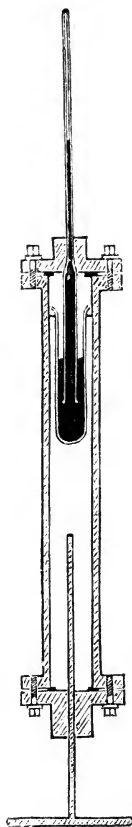
placed. The vessel *A* is heated over a fire, *B* being kept cool, when the ammonia distils off and condenses in *B* at a pressure depending on the temperature of *B*. If the temperature be  $0^{\circ}\text{C}$ ., the pressure will be about  $5\frac{1}{2}$  atmospheres. When nearly all the ammonia has distilled over, the communication between *A* and *B* is closed by turning the tap, and the vessel *A*, which now contains nearly pure water, is allowed to cool. The material to be frozen is then placed in the cavity within the vessel *B*, and when the water in *A* is sufficiently cold, the communication between *A* and *B* is reopened, and the liquid ammonia in *B* boils away rapidly, absorbing the latent heat of evaporation, while the gas is absorbed by the water in *A* as rapidly as it is produced. The rapid absorption of the latent heat of evaporation by the ammonia in *B* causes the reduction of temperature which it is the object of the apparatus to produce.

217. The temperatures and pressures at which several gases have been liquefied are given in the following table:—

	Pressure in Atmospheres.	Temperature
Sulphurous Anhydride . . . . .	2 . . . . .	$7^{\circ}\text{C}$ .
Chlorine . . . . .	4 . . . . .	$15\cdot5^{\circ}$
Cyanogen . . . . .	4 . . . . .	$15\cdot5^{\circ}$
Ammonia . . . . .	$6\frac{1}{2}$ . . . . .	$10^{\circ}$
Sulphuretted Hydrogen . . . . .	17 . . . . .	$10^{\circ}$
Carbonic Anhydride . . . . .	36 . . . . .	$0^{\circ}$
Hydrochloric Acid . . . . .	40 . . . . .	$10^{\circ}$
Nitrous Oxide . . . . .	50 . . . . .	$7^{\circ}$

218. **Freezing Mixtures.** The action of freezing mixtures depends upon the latent heat absorbed by bodies in fusing or evaporating. If ice and salt be pounded together, the ice melts through the action of the salt, absorbing the latent heat of fusion, and if the materials be originally at  $0^{\circ}\text{C}$ ., a temperature of  $-20^{\circ}\text{C}$ . may be produced. The action of sulphuric acid on crystals of sodic sulphate is very similar. The rapid evaporation of liquids produces still greater degrees of cold. If two watch-glasses be moistened with water and placed under the receiver of an air-pump, by quickly exhaust-

FIG 31.



ing the receiver the glasses may be frozen together. The evaporation of water in air at ordinary pressure will serve to freeze small quantities of water if the temperature of the air be  $2^{\circ}\text{C}$ ., provided that it be perfectly free from vapour. By mixing solid carbonic anhydride with ether, and allowing the mixture to evaporate *in vacuo*, Faraday obtained a temperature of  $-110^{\circ}\text{C}$ . By causing a mixture of carbonic bisulphide and liquid nitrous oxide to evaporate *in vacuo*, Natterer obtained a temperature which he estimated at  $-140^{\circ}\text{C}$ .

219. **Andrews' Apparatus.** Dr. Andrews' apparatus for the compression of gases consists essentially of a tube of glass (Fig. 31), of which the upper portion has a very fine bore, while both the bore and external diameter of the lower part of the tube are considerably greater. This tube contains the gas to be compressed. The lower part of the tube is immersed in mercury contained in a test-tube which is suspended in a copper cylinder. The conical shoulder of the tube rests in a corresponding seat in the cover of the cylinder, which is screwed down upon a flange attached to the top of the cylinder, a leather washer covered with lard being placed between the grooved surface of the flange and cover. The copper cylinder is filled with water, and a slender steel screw entering at the bottom serves to diminish the capacity of the cylinder, and thus to produce the requisite pressure. On screwing up the steel screw the pressure of the water drives the mercury from the test-tube up the thermometer-tube, compressing the gas above it.

If the tube be filled with pure carbonic anhydride at  $13.1^{\circ}\text{C}$ ., when the pressure reaches about 48 atmospheres the gas begins to liquefy, the pressure remaining constant until the whole has assumed the liquid condition. On

increasing the pressure, the liquid will be seen sensibly to contract. On diminishing the pressure, the liquid again expands; and if the pressure be rapidly relieved, the liquid will boil violently, bubbles of gas being formed in its midst as soon as the pressure becomes sensibly less than 48 atmospheres. At  $21.5^{\circ}\text{C}$ . carbonic anhydride requires a pressure of about 60 atmospheres to liquefy it, while at temperatures above  $30.9^{\circ}\text{C}$ . no amount of pressure we can apply will reduce it to the liquid condition; but of this we shall speak shortly.

**220. Cailletet's Apparatus.** The apparatus with which Cailletet first succeeded in liquefying air differed only from that of Andrews just described in the manner in which the pressure was produced, and in having a stop-cock by which the water could escape from the cylinder, thus allowing the pressure to be suddenly relieved. In Cailletet's apparatus the cylinder was connected with a powerful force-pump, somewhat similar to the force-pump of a Bramah press, and the pressure applied by its means. The thermometer-tube containing the gas was cooled by a freezing mixture to about  $-29^{\circ}\text{C}$ ., and a pressure of about 300 atmospheres was applied, but under this pressure carbonic oxide, nitrogen, oxygen, and hydrogen retained the gaseous state. On suddenly relieving the pressure by turning the stop-cock, the rapid expansion of the gas occasioned so great a diminution of temperature that in the case of each of these gases a liquid spray was produced in the tube.

**221. Pictet's Apparatus.** Pictet worked on a somewhat larger scale than Cailletet. By the employment of double-action pumps, which served to exhaust the gas from one vessel and to compress it in another, Pictet liquefied a quantity of sulphurous anhydride in a large tube, and by pumping the gas out from one end of the tube, liquefying it in a cold vessel by pressure, and then causing the liquid to enter the other end of the tube, he kept up a constant circulation of the same sulphurous anhydride. In the interior of the sulphurous anhydride tube was placed another tube into which carbonic anhydride passed from the vessel in which it was generated. The rapid and

continued evaporation of the sulphurous anhydride under the diminished pressure kept up by the pumps cooled the carbonic anhydride so much as to liquefy it under the pressure to which it was exposed. The liquid carbonic anhydride then flowed into a tube, through the centre of which was passed a smaller tube, into which was forced the air or other gas to be liquefied. A second set of pumps removed the carbonic anhydride as it evaporated, and forced the gas so removed back into the tube immersed in the sulphurous anhydride, thus keeping up a constant circulation of the carbonic anhydride. A third set of pumps forced air or any other gas to be examined into the narrow tube immersed in the liquid carbonic anhydride which was rapidly evaporating under the action of the pumps. The tube containing the air (or other gas) was provided with a stop-cock by which the pressure could be suddenly relieved. The temperature of the gas, already cooled by the evaporation of the carbonic anhydride, became so much lowered by the sudden expansion that a liquid spray was produced in all the cases tried, and it was stated that hydrogen became solidified into a grey metallic substance.

**222. The Critical Point.** M. Cagniard de la Tour, in 1822, enclosed a quantity of alcohol in a tube so as to occupy about two-fifths of the volume of the tube. A small pellet of mercury separated the alcohol from the air, which at first filled the remainder of the tube, and the compression of this air served to measure the pressure to which the alcohol was exposed. On raising the temperature to about  $225^{\circ}\text{C}$ . (according to Cagniard de la Tour) the alcohol suddenly disappeared, or rather, the surface separating it from its vapour vanished, the pressure recorded being about 129 atmospheres. In another experiment, when the alcohol filled much more than two-fifths of the interior of the tube, an explosion took place, but when the volume of the alcohol was much less, it all evaporated before reaching the temperature at which the surface vanished in the first experiment. A similar result was obtained with ether, naphtha, and water; but in the case of water it was necessary to add a little sodic carbonate

to prevent the solution of the glass. These experiments were afterwards repeated and the results verified by Faraday, and more recently by Andrews. As the temperature is raised the bounding surface of the liquid becomes less and less curved, indicating a diminution of the surface tension, and at the same time less distinct, until it becomes perfectly plane just before it vanishes altogether. Immediately after the vanishing of the surface a flickering haziness is sometimes noticed in the tube, as if some of the contents alternately became liquid and then gaseous again. It seems that as the temperature is raised the vapour becomes more and more dense through increased evaporation, while the liquid expands, becoming less dense; so that the properties of the two gradually approach one another, and at length the whole becomes apparently one continuous mass. This temperature, at which the liquid and the gaseous states merge into one another, has been called by Andrews the *critical point*.

223. As the temperature is raised the latent heat of evaporation of liquids diminishes, indicating that less energy is required to effect the transformation of the liquid into the gas, and this diminution appears to continue without limit as the temperature approaches the critical point; so that when this temperature is reached, the latent heat of evaporation becomes *nil*, the liquid and gaseous states being in fact identical. When a substance is heated above the critical temperature, it seems to be impossible to liquefy it by pressure. Faraday was of opinion that  $-166^{\circ}\text{C.}$  is above the critical temperature for air, oxygen, hydrogen, nitrogen, carbonic oxide, and marsh gas. Air can be easily compressed to a density far exceeding that of water, without any apparent change taking place in its constitution.

224. Dr. Andrews has very carefully studied the behaviour of carbonic anhydride when near the critical point by means of his apparatus for the compression of gases described in Art. 219. By surrounding the thermometer-tube by a rectangular case with plate-glass front and back, and filling this case with water whose temperature could be very accurately adjusted, he has found the

critical temperature to be  $30.92^{\circ}\text{C}$ . In order to obtain a substance in the critical state, it is necessary not only to adjust the temperature to the critical point, but also the pressure to what may be called the critical pressure. These two conditions will determine the volume of any given quantity of the substance, and therefore its density; so that we may speak of the critical temperature, critical pressure, and critical volume of a substance. Hence the difficulty of observing the phenomena attending the passage of a substance through the critical point when sealed in a glass tube; for if the volume of the tube exceed the critical volume, the substance will entirely evaporate before reaching the critical temperature; while if it fall short of the critical volume, the liquid will expand till it fills the tube, and the substance will remain entirely in the liquid state until the tube bursts; or, if the tube be strong enough to resist the pressure, the liquid will pass by a continuous process into the gaseous state, as the critical point is reached, without any change taking place in its appearance, and the contents of the tube will not be partly liquid and partly gaseous, so as to show the bounding surface of the liquid when the critical point is reached.

225. The critical temperatures and pressures of some substances, as determined by Dr. Andrews, are given in the following table:—

	Temperature.	Pressure in Atmospheres.
Carbonic Anhydride . . . . .	$30.92^{\circ}\text{C}$ .	75
Ether . . . . .	$187.5^{\circ}$	37.5
Alcohol . . . . .	$258.7^{\circ}$	119
Carbon Bisulphide . . . . .	$262.5^{\circ}$	66.5
Water . . . . .	$411.7^{\circ}$	?

226. If the temperature of a substance be raised above the critical point, so that the whole becomes gaseous, and be then gradually lowered, the pressure being maintained above the critical pressure, the substance will pass *continuously* from the gaseous to the liquid state as the temperature passes through the critical point, no apparent change taking place in the nature of the contents of the tube;

but on sufficiently diminishing the pressure after the temperature has fallen below the critical point, the liquid will be seen to boil.

227. By placing the tube of Andrews' apparatus in a solar microscope in front of an oxyhydrogen or electric lamp, the radiation from the lamp suffices to raise the temperature of the tube above the critical point for carbonic anhydride, and at the same time to project an image of the contents (magnified say 100 diameters) on a screen suitably placed. A blast of air from a bellows serves to lower the temperature below the critical point when required, without interfering with the image on the screen. By this means all the phenomena above described as attending the passage of carbonic anhydride through the critical point can be exhibited.

228. **The Spheroidal State.** If a clean metal plate be heated to redness, and a drop of water placed upon it by means of a pipette, the drop will evaporate but slowly, provided the temperature of the metal be kept up; but if the metal be allowed to cool, the drop will presently pass into vapour with explosive violence. If the surface of the metal be slightly convex upwards, the drop can be easily held at the highest point of the surface by means of a cold wire inserted in the drop, but not touching the hot metal. It is then easy to see that the drop is not in contact with the metal surface, but is supported slightly above it, as shown in Fig. 32. That the drop is not in contact with the hot metal may also be shown by connecting the wire which is inserted into the drop with one pole of a battery, the other pole being connected through a galvanometer with the hot plate. In this case no deflection of the galvanometer will take place so long as the plate is sufficiently hot to support the drop as above described, but if the temperature be lowered till vapour is produced with explosive violence the galvanometer will at the same time be deflected, indicating that the drop has come into contact with the plate, which of course accounts for the sudden generation of steam. So long as the drop is supported without coming into contact with the hot metal,

FIG. 32.



its form is approximately spheroidal, though flatter at the lower than at the upper surface, and this condition of the drop is called the *spheroidal state*. Having been first carefully examined by Leidenfrost, it is frequently known as Leidenfrost's phenomenon.

229. All liquids which readily evaporate can be made to assume the spheroidal condition; but for a given metallic surface, the lowest temperature at which the spheroidal condition can be produced depends on the nature of the liquid and the pressure to which it is exposed. As a general rule, the more volatile the liquid the lower the temperature required; while for a given liquid, reduction of pressure allows diminution of temperature, since it facilitates evaporation.

The proximity of the heated metallic surface causes rapid evaporation to take place from the lower surface of the drop, and this rapid production of vapour from one surface only prevents the drop approaching the metal plate within a certain distance (for as it comes nearer the rate of evaporation must increase), in the same way as the expulsion of a stream of water from the stern of a ship propels the ship; or the expulsion of the shot from a gun causes the latter to kick; or again, the expulsion of the steam-jet from Hero's engine, or "the little marvel," produces rotation of the boiler. Any circumstance which accelerates evaporation of course facilitates the assumption of the spheroidal condition; but if the liquid once comes absolutely into contact with the metal, it becomes rapidly heated throughout, and boils with explosion.

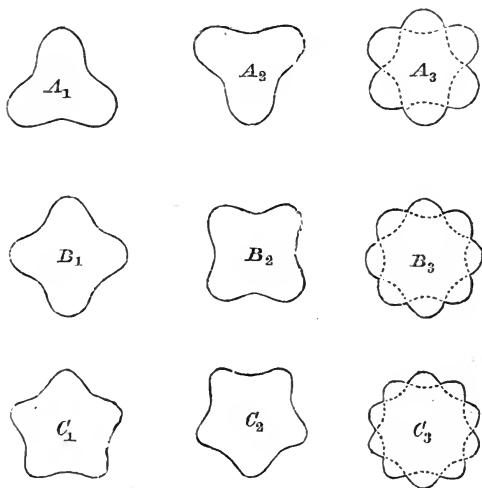
230. It was by availing himself of the spheroidal state of volatile liquids, that Faraday was able to freeze water and mercury in a white-hot platinum crucible. By rapidly injecting a mixture of liquefied sulphurous anhydride and ether into the hot crucible, the liquid was made to assume the spheroidal condition, so as to form a large "drop" without coming into contact with the vessel. Rapid evaporation quickly lowered the temperature of the drop sufficiently to freeze a small quantity of mercury held in a spoon near its centre. By employing a solution of solid carbonic anhydride in ether, in place of the



mixture first used by Faraday, the experiment is greatly facilitated.

231. If a tolerably large spheroid of water be formed in a clean platinum cup, it will be frequently seen to assume a beaded appearance, always exhibiting, however, an *even* number of beads around its circumference, as shown by the continuous lines in  $A_3, B_3, C_3$  (Fig. 33). On examining the drops carefully the outline of the beads will be seen to be continued in a fluted form within the drop, as shown by the dotted lines in  $A_3, B_3, C_3$ . If the

FIG. 33.



room be darkened and the drop illuminated by electric sparks, it will be seen to be constantly changing its form. Thus, if when examined by a continuous light, it present eight beads, as in  $B_3$ , when illuminated by the sparks it will present four beads and four flutes, sometimes appearing as  $B_1$  and sometimes as  $B_2$ . The drop is in this case vibrating like a bell which is sounding the second harmonic *above* the fundamental note, and therefore possessing eight vibrating segments, and the figure  $B_3$  seen by a continuous light is but the superposition of the images of the drop in its extreme vibration forms, shown at  $B_1$

and  $B_2$ . It is easy to obtain all the forms shown in Fig. 33, the decagon,  $C_3$ , being apparently an especial favourite, but generally the number of segments diminishes as evaporation reduces the size of the drop. Sometimes the drop will rotate about a vertical axis while vibrating, and this of course multiplies the apparent number of vibrating segments when viewed by a continuous light, but the sparks soon indicate the true state of the case.

232. Many accidental causes may serve to start vibration in a spheroid. Sometimes a quick but gentle touch with a hot wire will start them, but if the spheroid be left to itself, something generally happens to produce a vibration. Once set up, the production of vapour is a sufficient cause to sustain them; for more vapour will be produced from the projecting segments than from the others, and this vapour will escape from underneath them, and so press against their edges *as they recede*, while the escape of vapour past the edges of the advancing segments will be somewhat less. We thus have a periodical force and supply of energy acting in such a manner as to sustain the vibrations.

233. **Sublimation.** Many solids evaporate at temperatures below that at which they melt under ordinary pressure, and increase of pressure or diminution of temperature causes the vapour at once to return to the solid form. The direct passage of a vapour into a solid is called *sublimation*. Camphor, iodine, ammoniacal salts, and some other volatile solids frequently sublime upon the surfaces of vessels containing them. When aqueous vapour is condensed upon surfaces at a temperature below the freezing-point, *hoar-frost* is produced, which is consequently sublimed water.

234. **The Triple Point.** If a block of ice be placed in a vacuum at a temperature below the freezing point, it will evaporate until the pressure of the vapour attains a certain amount depending on the temperature. Regnault has investigated the pressure of aqueous vapour in the presence of ice through a range of several degrees below  $0^\circ \text{C}$ . By carefully cooling water below  $0^\circ \text{C}$ ., so as not to cause it to solidify, he also determined the

pressure of aqueous vapour in the presence of water for a few degrees below  $0^{\circ}\text{C.}$ , and found the result was not quite the same as when only ice was present. There is, however, a particular temperature at which water, ice, and aqueous vapour can exist within a closed vessel, no other substance being present in the enclosure. The necessary condition for such co-existence is obviously that the pressure of aqueous vapour at this temperature should be the same, whether it be in the presence of ice or water, and should be the pressure for which the existing temperature is the melting point of ice. This temperature is called the *triple point*; it is about  $0.007^{\circ}\text{C.}$

Some other properties of steam will be mentioned in Chapter XII.

**235. Changes in Water.** It may be of use briefly to trace the changes which take place in a mass of ice when heated.

Suppose we have a pound of ice at  $-20^{\circ}\text{C.}$  When heated it expands, its coefficient of expansion being about  $.00012$  per  $1^{\circ}\text{C.}$ , and its specific heat about  $.5$ . It will thus absorb about 10 units of heat, while its temperature is raised to  $0^{\circ}\text{C.}$  before it begins to melt. The substance will then remain at this temperature till it has absorbed about  $79.5$  units of heat, when it will all have been converted into water, and in the transition its volume will have contracted by about 9 per cent. The temperature will then go on increasing steadily, the water contracting in bulk till its temperature reaches  $4^{\circ}\text{C.}$ , after which it will continue to expand until about 101 units of heat have entered the water, when its temperature will be  $100^{\circ}\text{C.}$ , and its volume will have increased by rather more than 4 per cent. of its volume as water at  $0^{\circ}\text{C.}$  The water will then begin to boil, and if the heat be continued, the whole will pass off as steam, absorbing in the process about 537 units of heat, and expanding to about 1690 times the volume of the water at  $0^{\circ}\text{C.}$ , the barometer being supposed to stand at 30 inches. Thus, the amount of heat absorbed in the transformation of a pound of ice at  $-20^{\circ}\text{C.}$  to steam at  $100^{\circ}\text{C.}$  is—

$$10 + 79.5 + 101 + 537 \text{ units} = 727.5 \text{ units,}$$

or, as much as would raise 727·5 lbs. of water through 1° C.

236. **Theories of Evaporation.** The earliest theories with which we are acquainted assumed that vapour consisted of a compound of water with fire, which rendered the vapour light and caused it to ascend. When particles of vapour were blown together, the fire particles were shaken off, and the water fell as rain, while the fire sometimes made its appearance as lightning.

The supporters of another theory held that vapour consisted of vesicles, or little balloons of water filled with an *aura*, which was specifically lighter than air, and caused the vesicles to ascend.

Others again held that the air was the cause of evaporation, and that the air held aqueous vapour in chemical solution. This theory was disproved by De Saussure, who showed that aqueous vapour was produced in the vacuum of an air-pump to the same extent as in the air.

## CHAPTER VII.

EFFECTS OF HEAT UPON THE MECHANICAL, MAGNETIC, AND ELECTRICAL PROPERTIES OF BODIES. THERMO-ELECTRICITY. PELTIER EFFECT. THERMO-PILE.

237. We have seen (Art. 72) that a piece of india-rubber stretched by a weight will contract if heated. Hence it appears that in the case of india-rubber the elasticity is increased by raising the temperature. The reverse is the case with the majority of bodies, which become less and less elastic as their temperature is raised, till at length they become viscous fluids. The ultimate strength of materials is also affected by change of temperature. Wrought iron is said to have the greatest strength to resist tension near the boiling point of water. When the temperature is raised much above this, the strength of the iron rapidly diminishes. It is for this reason that both wrought and cast iron are almost useless *as weight bearers* in the construction of fireproof buildings. Gun metal, consisting of 88 per cent. of copper, 10 per cent. of tin, and 2 per cent. of zinc, has its tensile strength diminished to one-half at a temperature a little above  $120^{\circ}$  C., when it presents a highly crystalline fracture. This does not occur when no zinc is present, a fact which should be borne in mind in the construction of boiler fittings.

238. **Annealing and Tempering.** By raising solid bodies to a high temperature and then slowly or rapidly cooling them, their physical constitution may sometimes be changed considerably. By very slow cooling from a high temperature some bodies are rendered soft and tough, while they become hard and brittle if cooled rapidly. Tool steel may be made very soft and pliable

by heating it to redness, and cooling it very slowly ; but if cooled from a red heat by plunging it into water, it becomes sufficiently hard to scratch glass. Steel is generally *tempered* by first hardening it, and then heating it to a certain temperature, depending on the degree of hardness required, and allowing it to cool rapidly from this temperature. The higher the temperature to which the steel is raised in the second heating (such temperature being kept considerably below that to which it was previously raised to harden it), the softer is the steel. If a piece of steel exposing a bright and clean surface be heated in air, a layer of oxide is formed upon it, which exhibits the colours of thin films. The thickness of this layer, and therefore the colour presented, depends on the temperature. In tempering steel, after hardening it by heating it to bright redness and cooling it suddenly in water, a small portion of the surface is made bright, and the temperature acquired in the second heating is determined by the tint exhibited. At about  $220^{\circ}\text{C}$ . a faint straw-colour begins to appear. This deepens into yellow at about  $245^{\circ}\text{C}$ ., and brown at  $260^{\circ}\text{C}$ . As the temperature is increased, the brown gradually shades off into purple ; and the surface becomes blue at about  $320^{\circ}\text{C}$ . The edges of tools for cutting metal are tempered at a straw-colour ; cutting-tools for wood, knives, etc., at a brown ; springs, saws, and instruments requiring great flexibility, at a purple or blue. In tempering chisels, turning-tools, etc., it is convenient to heat the end remote from the edge, allowing the heat to be conducted towards the edge, and plunging it in water as soon as the edge reaches the proper tint. By this means the edge becomes the hardest portion of the instrument.

**239. Unannealed Glass.** If glass be rapidly cooled from a high temperature, it becomes very brittle, and the slightest scratch upon its surface will frequently occasion its fracture. This liability to fracture is in great part due to a state of strain developed in the glass through the rapid cooling. Rupert's drops consist of pear-shaped drops of glass which are allowed to fall in a melted state into cold water. The exterior portion thus becomes solidified,

while the interior is still at a red heat. As the interior cools, the glass tends to contract, but is kept stretched by the outer envelope, which has become rigid. If the interior be disturbed by breaking off the *tail* of the drop, this state of strain is relieved, and the glass crumbles to powder with explosive violence. The state of strain into which glass is thrown by rapid cooling is made manifest by examining it with polarised light, when the *unannealed* glass produces colours like thin plates of mica or other doubly refracting crystals. Glass is annealed by cooling it very slowly in an annealing oven.

240. **Bologna flasks** are small flasks or bottles which are allowed to cool rapidly after having been blown. By this means the exterior portions become solid before the interior have cooled to the same temperature, and as the interior portions subsequently harden and contract, they are thrown into a state of tension by their cohesion to the external layers. Such flasks will stand a great deal of violence so long as the interior surface is not scratched, but the slightest scratch by a grain of sand on the internal surface causes the flask to fly to pieces. Injury to the outside of the flask is like a sawcut made in the upper surface of a beam supported at the ends and loaded in the middle. Injury to the interior is like a cut made in the lower surface of the same beam, and, like a little rift on the edge of a piece of paper under tension, not only diminishes the available section of the material, but concentrates the tension around the end of the rift and leads to inevitable destruction. The exterior layers of the flask are subject to pressure, like the upper layers of the beam; the interior layers to tension. The strains frequently produced in iron castings have already been mentioned.

241. **Copper and Lead.** Copper, when heated, behaves differently from steel, since it is rendered soft and malleable by heating it to redness and cooling it suddenly in water. Continual hammering renders copper and brass hard and brittle, and in shaping these metals under the hammer it is necessary frequently to anneal them by heating them and then cooling them quickly. If the copper be required hard, in order to resist rubbing, it is

best to subject it to the hammer after it has been heated for the last time.

Lead, when heated to between  $100^{\circ}\text{C.}$  and  $200^{\circ}\text{C.}$ , is softened thereby; and in making sharp bends in lead pipes, it is desirable to warm the interior, or throat, of the bend in order to facilitate the compression of the lead, and prevent the excessive extension and thinning down of the metal which would otherwise take place on the outside, or heel, of the bend, where the greatest thickness is required, if the bend is to last as long as the rest of the pipe. If lead is heated to a temperature much above this, it is hardened and rendered brittle.

**242. Temperature and Magnetism.** The relations of many bodies to magnetism and electricity undergo considerable variation as their temperature is changed. Thus, steel magnets become weaker as their temperature is raised, but if the range through which they are heated be only a few degrees from ordinary temperatures, they very nearly recover their original strength on cooling. If, however, a steel magnet be heated to redness, it completely loses its magnetism, and does not recover it as it cools.

For very small magnetising forces, the *permeability* of iron increases greatly with the temperature, but rapidly falls at about  $700^{\circ}\text{C.}$ , becoming no greater than that of the air at about  $760^{\circ}\text{C.}$  For very great magnetising forces the permeability diminishes steadily as the temperature increases.

**243. Electrical Resistance.** Different bodies offer different resistances to the passage of an electric current through them. The resistance of any conductor is generally expressed in terms of that of a certain standard wire at a defined temperature. It has been found that the resistance of all bodies which conduct electricity, like metals, without themselves undergoing any change, is increased by increase of temperature; but that the resistance of bodies in which the passage of electricity is accompanied by chemical decomposition, that is, of *electrolytes*, is diminished by increase of temperature. Glass belongs to the latter class; and if two pieces of metal be



connected with the electrodes of a Holtz's machine, and united by a few inches of glass rod, when the glass is red hot the electricity passes quietly through it. As the glass cools, the passage of the electricity is accompanied by a beautiful glow, very apparent in a darkened room, the colour of the light depending on the character of the glass; but long before the glass has cooled to the ordinary temperature, the electricity ceases to pass through it to any sensible extent, and prefers to make its way as a "spark" through the air. The resistance of carbon also diminishes as the temperature is raised. The carbon filaments of glow lamps have about twice as much resistance when cold as when hot.

244. As a general rule, the electric resistance of pure metals changes more for a given change of temperature than that of alloys. It has been found that the resistance of metals increases pretty uniformly with the temperature, and this property has been utilized for the purpose of determining the temperature of furnaces. The electric pyrometer consists of a fine coil of platinum wire connected to terminals whose resistance is very small compared with its own. This coil is placed in the interior of the furnace, and a current being sent through it its resistance is measured in the usual manner; and its resistance and rate of increase per degree at ordinary temperatures being known, the temperature to which it is exposed can be determined.

245. The increase of the electrical resistance of iron with rise of temperature is much greater than that of other metals. This makes iron resistance coils of especial value in order to steady the current in some cases of electric lighting. For instance, in the case of an arc lamp in which the length of the arc is fixed, if the current increase the temperature will rise, and this will occasion diminution of resistance, tending to further increase of the current, so that the arrangement is unstable; and such lamps will not burn in parallel circuit without special arrangements for controlling the current. The introduction of a coil of fine iron wire in the circuit will sometimes obviate the difficulty; and although it involves waste of

energy in heating the wire, it may sometimes be economical when the lamp is used only occasionally or in temporary installations. If the current increase, the resistance of the lamp will be diminished; but if the coil of iron wire be properly constructed, owing to increase of temperature the resistance of the wire will be increased more than that of the lamp will be diminished, and the resistance of the two together will be increased. The increase of resistance will tend to reduce the current, and similarly if the current should fall below its normal value, the resistance of the current will diminish, and the current will tend to increase again, so that the arrangement is *stable*. To effect the same adjustment by wires of any other material than iron would necessitate far too great a resistance, and consequent waste of energy in the wire.

246. If an electric current be sent through a conductor, heat is produced, the amount generated per second being proportional to the resistance of the conductor, and to the square of the current conjointly. Hence a strong current will quickly raise to a white heat a fine platinum wire whose capacity for heat is small. A wire which can be so heated is very convenient for many purposes, since it may be placed when cold in positions in which it is impossible to place a heated body, and may be subsequently heated when required. A platinum wire heated to redness by a current from a battery is sometimes employed by surgeons for cutting and cauterising.

Incandescent electric lamps consist of specially prepared carbon filaments made from cotton, bamboo fibre, or other material, which are placed in small glass globes exhausted of air as far as possible by mercury pumps. The ends of the carbons are cemented, electrotyped, or otherwise connected to platinum wires fixed into the glass and terminating on the outside in loops which form the electrodes of the lamp. When an electric current of sufficient strength is sent through the filament, it is raised to a bright white heat.

247. **Thermo-Electricity.** When an electric current passes from one metal to a different metal, heat is

produced or absorbed at the junction, according to the direction of the current and independently of the resistance of the metals. The rate at which heat is produced or absorbed in this action is simply proportional to the current and not to its square. This phenomenon was first noticed by Peltier, and is known as the Peltier effect. It may be readily rendered manifest by soldering a small bar of bismuth and one of antimony together, and placing the junction within the bulb of an air thermometer. If the current from a single Grove's cell be passed through the bar, heat will be produced when the current passes from antimony to bismuth, and cold when it passes in the opposite direction. Too strong a battery should not be used, or the heat due to the simple resistance of the metals will conceal the Peltier effect.

248. If two different metals be soldered or twisted together at each end, so as to form a continuous circuit, no current of electricity will pass round it, provided that both the junctions are at the same temperature; but if one junction be at a higher temperature than the other, a current will generally be produced. In the last article we stated that a current of electricity passing from one metal to another generally heats or cools the junction. In the case here contemplated the current generally passes in such direction as to cool the hot and heat the cold junction, thus tending to equalise the temperature of the circuit. It is clear that it cannot pass in the other direction, as in that case a current once started in a circuit of two metals originally of uniform temperature would increase indefinitely, heating one junction and cooling the other at a continually increasing rate.

249. In the following list the metals are so arranged that if any two of them be connected to form a circuit at ordinary temperature, one junction being slightly hotter than the other, the current will flow across the hot junction from the metal which stands higher in the table to the other. Thus, if the metals be antimony and bismuth, the current will flow from bismuth to antimony across the hot junction. If they be copper and iron, it will flow from copper to iron across the heated junction; and if

they be bismuth and iron, it will flow from the bismuth to the iron across the heated junction.

Thermo-electric Arrangement of Metals.

- |              |               |
|--------------|---------------|
| 1. Bismuth.  | 7. Gold.      |
| 2. Mercury.  | 8. Silver.    |
| 3. Lead.     | 9. Zinc.      |
| 4. Tin.      | 10. Iron.     |
| 5. Copper.   | 11. Antimony. |
| 6. Platinum. |               |

The position of a metal upon the list frequently depends upon its purity and mechanical condition.

250. If a circuit be formed of copper and iron, and a galvanometer be introduced by cutting the copper wire and connecting the ends with the electrodes of the galvanometer, and if one of the junctions be heated, it will be seen that the current increases until a temperature of about  $260^{\circ}\text{C}$ . is reached; when, on heating the junction still more, the current diminishes, and when the hot junction is as much above  $260^{\circ}\text{C}$ . as the cold junction is below this temperature, the current ceases. On raising the temperature still higher, the current is reversed in direction, and we have *thermo-electric inversion*, the current passing from iron to copper across the hot junction. The temperature at the hot junction when the current begins to diminish is called the neutral temperature of iron and copper. Professor Tait has shown that the electro-motive force in a thermo-electric combination of two metals is proportional to the product of the difference of temperature of the junctions and the difference between the mean of the temperatures of the junctions and the neutral temperature of the two metals. Thus, in the case of copper and iron, if the hot junction be at  $220^{\circ}\text{C}$ . and the cold at  $20^{\circ}\text{C}$ ., the electro-motive force is proportional to

$$(220 - 20) \left( 260 - \frac{220 + 20}{2} \right),$$

or  $200 \times 140$ , while if the hot junction be at  $600^{\circ}$  and the cold at  $40^{\circ}\text{C}$ ., the force is proportional to

$$(600 - 40) \left( 260 - \frac{600 + 40}{2} \right),$$

or to  $560 \times (-60)$ , the negative sign in this case indicating that the direction of the current has been reversed.

251. If one junction of a copper-iron couple be at the neutral temperature and the other junction below the neutral temperature, a current will flow in the circuit, the cold junction will be heated, but the hot junction will not be cooled, as at the neutral temperature the two metals behave like one. The heat must therefore be absorbed from the iron or copper or both. This was predicted by Sir William Thomson, who found experimentally that when heat flows from *cold to hot in copper*, or from *hot to cold in iron*, it cools the metal, and heats it when it flows in the opposite direction. This is known as the Thomson effect.

252. Thermo-electric combinations are sometimes employed for determining the temperature in situations where ordinary thermometers cannot be used. Thus, the temperature at the bottom of the sea may be determined by employing long wires of copper and iron in circuit with a galvanometer. One of the junctions is caused to descend to the sea bottom, and the other placed in a bath whose temperature is adjusted until there is no current through the galvanometer. The temperature of the bath must then be the same as that of the sea bottom, and can be read by an ordinary thermometer.

253. If several bars of bismuth and antimony are united, as in the figure, and the alternate junctions heated, the electro-motive force in the circuit is the sum of the forces due to each pair of consecutive junctions, and by using a large number of bars a very sensitive instrument may be constructed. The thermo-electric pile consists of a number of such bars, sometimes united in the form of a cube, the alternate junctions being on opposite faces. If the extreme bars are connected by a wire passing round a freely suspended magnetic needle, any difference in the temperatures of the opposite faces of the cube at which the junctions are situate, produces a corresponding deflection of

FIG. 34.

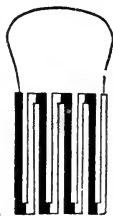
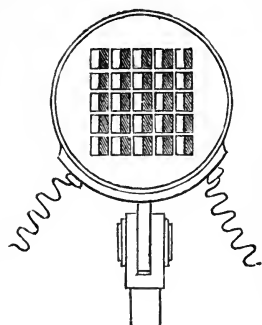


FIG. 35.



the needle, the pile thus acting as a very delicate differential thermometer. One of the faces of such a pile is shown in Fig. 35.

Various forms of thermo-electric piles have been constructed in order to obtain strong electric currents for electrotyping, electro-plating, driving electro-magnetic engines, and other similar purposes; but in all the very powerful thermo-electric combinations which have hitherto been tried, one at least of the metals or alloys employed is very brittle, and liable to crack through rapid changes of temperature or through slight mechanical violence, and unless used with very great care they are soon rendered inefficient.

## CHAPTER VIII.

### ON THE TRANSMISSION OF HEAT.

254. THERE are three methods generally enumerated by which heat may be transmitted from one point to another. They are,—

I. *Conduction.*

II. *Convection.*

III. *Radiation.*

*Heat is said to be transmitted by conduction when it passes from hotter to colder portions of the same body, or from a hot body to a colder body in contact with it: the heat in this case being transmitted from each particle of the material of the body to contiguous particles in directions in which the temperature decreases. A familiar example of conduction is found in the transmission of heat along a poker, one end of which is placed in the fire.*

*Heat is transmitted by convection when the material body containing the heat is carried from one point to another.*

Thus, if hot water is carried in a bucket or conveyed in pipes from one point to another, the heat contained in the water is said to be transmitted by convection.

*Heat is said to be transmitted by radiation when it passes from one point to another irrespective of the temperature of the medium through which it passes.*

Thus, we feel the sun's rays to be warm, and a thermometer exposed to them will indicate a temperature considerably above  $0^{\circ}\text{C.}$ , though the air through which the rays pass may be at a temperature below the freezing point. Bodies which, like air, allow of the passage

of radiation through them, without themselves becoming heated, are called Diathermanous; while bodies which, like the metals, do not possess this property, are called Adiathermanous.

255. This last mode of transmission of heat differs from the two preceding, inasmuch as the heat does not pass from one point to another *as heat*, but is converted into another form of energy, which is only reconverted into heat under special circumstances; and in some cases is capable of causing the sensation of light if allowed to enter the eye, or even of producing a photographic picture on a properly prepared surface, by inducing chemical or molecular changes in the constitution of the body on which it falls. The transmission of heat by radiation is therefore a case of double *transformation of energy*, and may be classed with several other ways in which heat appears to be communicated from one place to another, though, during its passage, it is really not heat at all; as for example, when some of the heat generated in the furnace under the boiler of a steam-engine is converted into the energy of motion of the parts of the machinery, and partially reconverted into heat by the friction of the bearings, etc.; or, when the heat applied to one set of junctions in the thermo-electric pile is converted into the energy of an electric current, and reconverted into heat at the opposite set of junctions, and in the wire through which it passes. The phrase "radiant heat" is therefore unscientific, and it is best to employ the term *radiant energy* instead.

256. **Conduction of Heat.** In the distinctions made in the last article, attention is drawn to the fact that when heat passes by conduction, it always flows from hotter to colder portions of matter, existing always as heat in the particles of the body through which it travels. Radiant energy moves with the velocity of light, and subject to the same laws as light (see Art. 232), while the same beam may pass through various bodies differing in temperature to any extent and arranged in any order. It is not in fact taken up by the material particles of the



bodies through which it passes, but is supposed to consist of transverse vibrations of the luminiferous ether. If the material particles of the body do take up the energy of the vibrations, and become heated thereby, we have a case of absorption, and the energy so taken up ceases to be transmitted by radiation.

257. If we enter a very cold room, and touch various articles in the room, we find that the metal articles feel coldest of all, and of these we may notice that large masses of silver or copper feel especially cold to the touch, while the wooden furniture produces the sensation of cold in a less degree, and the hearth-rug or other woollen materials, hardly seem cold. Now, if a thermometer be brought into contact with all these articles in succession, it will register the same temperature, the reason of the difference to the touch being that the metallic bodies transmit heat through their own masses, and so take it away from the hand, much more readily than wood, and wooden articles much more readily than woollen materials. In fact, the bodies which feel colder than others, seem so, not because they are at lower temperatures, but because they are better conductors of heat. In high latitudes, where the cold is very intense, contact with a piece of metal exposed to the weather will inflict a blister on the hand.

For a reason similar to the above, in the hot room of a Turkish bath, metallic objects feel much hotter than others, and will inflict a burn, so that spectacles with metal frames have to be dispensed with. The human body is prevented from being unduly heated when exposed to a high temperature by the evaporation going on from its surface.

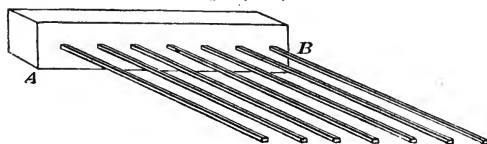
258. From what we have just said, we see that different bodies possess different powers of conducting heat, or different *thermal conductivities*.

The fact that there is a difference in the thermal conductivity of different substances may be readily shown by taking three teaspoons, made of silver, lead, and bone respectively, and placing their bowls in hot tea. The

handle of the silver spoon soon becomes too hot to touch without inconvenience, and in the same time that of the leaden spoon becomes hot, but less so than the former; while the handle of the bone spoon is only slightly heated. Silver spoons may be distinguished from those made of other metals, and plated, by the readiness with which they transmit the heat of the tea to the hand. It is on account of the low conductivity of such substances as wood, bone, glass, etc., that they are employed as handles for vessels containing hot bodies, or inserted between the handles and the vessels themselves. The value of blankets depends upon the extremely low conducting power of woollen materials, and a blanket will serve equally to keep a warm body from cooling or to keep ice from melting. The low conductivity of felt is utilized in the Norwegian cooking pot, which is a box lined with many layers of felt. The saucepan containing the food is first heated, and then placed in the box and shut in, when it will retain for a long while a temperature sufficiently high to carry on the operation of cooking.

259. The *measure* of the thermal conductivity of a substance will be defined in Art. 263. The relative conductivities of different substances may be compared by means of the arrangement shown in Fig. 36, and known

FIG. 36.

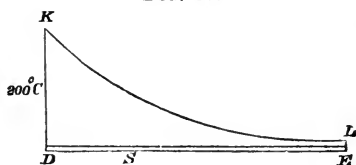


as Ingenhouz's apparatus. This consists of a trough *AB* containing oil, which can be heated to a high tempera-

ture. Equal bars of the solids whose thermal conductivities are to be compared are taken, and one end of each is passed through one side of the vessel *AB* into the oil. The surface of each bar is coated with a thin layer of wax, so that each exposes the same surface to the air, and will therefore cool at the same rate, other things being the same. Now, if the oil be heated to, say,  $200^{\circ}\text{C.}$ , and sustained at that temperature, the bars will gradually become hot, and the heat will travel towards their ex-

tremities, which are distant from  $AB$ , the wax melting at increasing distances along each bar. After a time, which will be different for the different bars, it will be found that the boundaries between the melted and solid wax no longer travel towards the ends of the bars, but remain stationary, however long the temperature of the oil may be maintained. The temperature of every point of the bar then remains unaltered, and if  $DE$ , Fig. 37, represent the bar, and  $S$  any point upon it, this will be the case when the rate of decrease of the temperature at  $S$  as we go towards  $E$  is such that the flow of heat across the section of the bar at  $S$  is equal to the rate at which heat is dissipated by the portion of the bar beyond

FIG. 37.



$S$ . The temperature of the bar will decrease in going from  $D$  to  $E$ , at a rate diminishing as the distance from  $D$  increases. If at every point of the bar a line be drawn at right angles to its length, and proportional to the excess of the temperature of the bar at that point above that of the air, the extremities of these lines will lie on a curve similar to  $KL$ . Now, the wax on the bars will be melted to distances where the temperature of the bars is the melting point of wax, and it may be shown that when the wax ceases to melt any farther, if all the bars are of the same section, the squares of these distances are proportional to the thermal conductivities of the bars. Hence, we have only to measure from the vessel of oil the distance on each bar along which the wax is melted, in order to compare the thermal conductivities of the materials of which the bars are composed. The bars are generally inserted into small tubes which pass through the oil across the box, and are soldered to each side of the box, so that the tubes are surrounded by oil, just as the tubes in a locomotive boiler are surrounded by water.

260. It should be borne in mind that the thermal conductivities of different bars cannot be determined by heating one end of each to the same temperature, and

observing the time required for points on all the bars at the same distance from the heated ends to be raised to a given temperature. For instance, in the experiment just described, we may observe the time required for the wax to be melted on each bar at a distance of three inches from the trough, after pouring the hot oil into the latter. We cannot, however, in this way compare the thermal conductivities of the bars because their capacities for heat per unit of volume may be different; thus, of two bars *A* and *B*, *A* may have the greater conductivity, but if its capacity for heat per unit of volume be also greater than that of *B*, the time required to melt the wax on *A* may be greater than that required to melt it on *B* to the same distance. In order to compare the thermal conductivities, we have therefore to wait until the temperature of any point of the bars has ceased to change, or till the flow of heat has become "steady," as explained in Article 262.

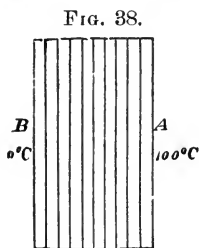
261. The statement made in the last Article can be well illustrated by simultaneously placing a bar of iron and a bar of bismuth, of the same dimensions, and each about 1 in. in length, upon end on a hot copper plate, a small piece of wax or paraffin having been previously placed on the upper end of each bar. (Care should be taken that the temperature of the copper is insufficient to melt the bismuth.) If the bars be not too long, and the temperature of the hot plate be sufficiently high, the wax or paraffin upon the bismuth bar will begin to melt before that upon the iron bar, but the rate at which the melting will progress after it has commenced will be much greater on the iron than on the bismuth; so that if the quantity of wax or paraffin is large, that on the iron will be completely melted before the other. Now, the thermal conductivity of iron is more than six times that of bismuth, yet the upper end of the bismuth becomes first heated to the melting point of wax or paraffin because the specific heat of iron is nearly four times that of bismuth; but after that the iron has become sufficiently heated to commence the fusion, heat is conducted along it much more rapidly than along the bismuth.

If the bars be of considerable length, and be coated all

over with paraffin, it will be seen that the paraffin will melt at first more quickly on the bismuth bar, but after it has melted on the bismuth bar for perhaps an inch, that on the iron bar will overtake it; and finally, the wax on the iron bar will be melted over more than twice the distance of that on the bismuth bar.

The thermal conductivity, divided by the capacity for heat per unit volume, determines the rate of transmission of temperature through the substance, and is called its *thermal diffusivity*.

262. If one side of a plate of metal be kept in contact with water, whose temperature is sustained at  $100^{\circ}\text{C}$ ., while the other side is kept in contact with melting ice, heat will flow through the metal by conduction from the hot to the cold side, and after a short time the metal will acquire its permanent condition with regard to temperature under these circumstances. When this is the case, the flow of heat through the metal will be the same at all parts, and continue constant so long as the temperature of the faces is kept unaltered. The flow is then said to be *steady*. *The flow of heat across the unit of area of any plate of given material is found to vary inversely as the thickness of the plate, and directly as the difference between the temperatures of the two faces.* Now, suppose the thick plate  $AB$ , Fig. 38, to be made up of a number of thin plates of equal thickness, as in the figure. Then, when the flow of heat is steady, the amount of heat which crosses each of the thin plates in a second is the same, and therefore the difference between the temperatures of the opposite faces must be the same for each plate, and the temperature of the thick plate must decrease uniformly from  $100^{\circ}\text{C}$ . on the one side to  $0^{\circ}\text{C}$ . on the other side.



If we take two plates of the same material and thickness, and suppose no heat to escape at the edges, if one side of each be kept at  $100^{\circ}\text{C}$ . and the other at  $0^{\circ}\text{C}$ ., the amount of heat which will flow through each of

the two plates in a given time will be obviously proportional to their areas.

263. DEF. *The number of units of heat which flow through the unit of area of a plate of unit thickness in the unit of time when the flow of heat is steady, the difference in the temperatures of the sides of the plate being  $1^{\circ}\text{C.}$ , is taken as the measure of the thermal conductivity of the substance of which the plate is composed, and is called its specific thermal conductivity.*

If the unit of length be one inch, and the unit of time one second, the thermal conductivity of a substance will be measured by the number of units of heat which flow in one second across each square inch of a plate whose thickness is one inch when the temperature of one side is kept  $1^{\circ}\text{C.}$  above that of the other. If the difference of temperature be  $10^{\circ}\text{C.}$ , the flow of heat will be 10 times this amount, and so on; while if the thickness of the plate be increased in any ratio, the flow will be diminished in the same ratio.

264. The thermal conductivities of some bodies have been measured by determining how much ice can be melted in a given time through a plate of known area and thickness, when one side is kept in contact with boiling water.

This method, however, leads to very fallacious results in the case of metals and other good conductors, for the heat is transmitted through the substance of the plate far more quickly than it can be given up to the water. The hot water on the one side gives heat to the metal, and the layer of water in contact with the metal becomes cooled, while the heat is transmitted so rapidly through the metal, that this layer becomes very much cooled before it can get away and be replaced by hotter water. Similarly, on the other side of the plate the layer of water close to the metal becomes heated, and receives heat so fast that its temperature is raised through several degrees before the heated layer gives up its place to a colder one. Hence the water actually in contact with the hot face of the metal, and therefore the hot face of the metal itself, will be much below the boiling point, while the water in con-

tact with the cold face, and therefore also the cold face itself, will be much above the freezing point; so that instead of a difference of temperature of  $100^{\circ}$  C. between the faces of the plate, it may happen that this difference is only  $3^{\circ}$  or  $4^{\circ}$ . The measure of the conductivity obtained will therefore be far too small. It is for this reason that copper fire-boxes do not sensibly increase the evaporative power of boilers. With bad conductors, however, the method may lead to fairly accurate results.

265. Suppose that a plate of wrought iron of 1 inch in thickness and 2 square feet in area is placed so as to form a division between water on the one side, which is kept at  $12^{\circ}$  C., and melting ice on the other side, and suppose that at the end of an hour we find that 164 pounds of ice have been melted. The amount of heat required to melt 164 pounds of ice at  $0^{\circ}$  C. is about 12956 units :

$\therefore$  in 1 hour through 288 sq. ins. of the plate, when the temperatures of its sides differ by  $12^{\circ}$  C., 12956 units of heat flow ;

$\therefore$  in 1 hour through 288 sq. ins. of the plate, when the temperatures of its sides differ by  $1^{\circ}$  C.,  $\frac{12956}{12}$  units of heat will flow ;

$\therefore$  in 1 hour through 1 sq. in. of the plate, when the temperatures of its sides differ by  $1^{\circ}$  C.,  $\frac{12956}{12 \times 288}$  units of heat will flow ;

$\therefore$  in 1 second through 1 sq. in. of the plate, when the temperatures of its sides differ by  $1^{\circ}$  C.,  $\frac{12956}{12 \times 288 \times 3600}$  units of heat will flow.

The measure of the thermal conductivity of wrought iron referred to a second and an inch as units of time and space, the unit of heat being taken, as usual, to be the amount required to raise 1 lb. of water  $1^{\circ}$  C. from  $0^{\circ}$  C., is therefore

$$\frac{12956}{12 \times 288 \times 3600} = \cdot 001....$$

Taking a centimetre to be two-fifths of an inch and a gramme to be  $\cdot 0022$  lb., and remembering that a cubic centimetre will offer a sectional area of  $\frac{4}{25}$  inch for the passage of the heat, while its thickness is  $\frac{2}{5}$  inch, it follows that the absolute conductivity of iron referred to the second, centimetre, and gramme as fundamental units will be

$$\frac{\cdot 001 \times \frac{2}{5}}{\cdot 0022} = \frac{2}{11} = \cdot 1818.$$

266. As another example, we may find how much water at  $100^{\circ}$  C. can be evaporated per hour at atmospheric pressure in an iron boiler which exposes a surface of 20 square feet to the fire, supposing the iron to be  $\frac{1}{2}$  inch in thickness and the fire to keep its lower surface at a temperature of  $150^{\circ}$  C.

The thermal conductivity of iron being  $\cdot 001$  referred to an inch and a second, it follows that the number of units of heat passing through the iron in an hour will be

$$\cdot 001 \times 2 \times 144 \times 20 \times 3600 \times 50,$$

since the difference of temperature of its sides is  $50^{\circ}$  C.; and since the latent heat of steam is represented by 537, the number of pounds of water evaporated will be

$$\frac{\cdot 001 \times 2 \times 144 \times 20 \times 3600 \times 50}{537} = 1931,$$

or the quantity of water evaporated per hour will be about 193 gallons. This is very much above the evaporative power of a boiler having only 20 square feet of heating surface, so that the difference of temperature between the surfaces of the plates must be much less than  $50^{\circ}$  C.

### 267. Conduction of Heat through Boiler Plates.

In ordinary practice with Lancashire boilers, the amount of water evaporated per square foot of heating surface per hour is not more than 6 or 7 lbs., taking the average throughout the boiler. Over the crown of the furnace the rate of evaporation is much greater than this, while



it must be less for the portion of the shell of the boiler which is exposed to the furnace gases in the side flues. This corresponds to a difference of temperature between the two sides of the plates of only three or four degrees for a half-inch plate. As an approximate rule, it is useful to remember that for a boiler plate three-quarters of an inch thick, a difference of temperature of one degree between its surfaces will correspond to an evaporation of 1 lb. of water per square foot per hour from the boiling point. If the plate is only half an inch thick, the rate of evaporation for the same difference of temperature between the surfaces will, of course, be one and a half times this amount; and so on.

268. We may suppose that in a Lancashire boiler working with natural draught and burning about 20 lbs. of coal per square foot of grate surface, the *average* difference of temperature between the furnace gases on the one side of the plates, and the water on the other side, taken throughout the boiler, is not less than  $600^{\circ}\text{C}$ ., and in many cases it is much higher. If the average thickness of the plates be five-eighths of an inch, and the rate of evaporation is 7 lbs. of water per square foot per hour, the temperature of the feed water being  $50^{\circ}\text{C}$ ., it follows that the difference of temperature between the surfaces of the plates is only  $6^{\circ}\text{C}$ ., or one per cent. of the average difference of temperature between the furnace gases and the water. It follows, therefore, that ninety-nine per cent. of the resistance which the heat meets with in passing from the furnace gases into the water lies outside the boiler plates. Doubling the thickness of the plates only changes their resistance from one per cent. to two per cent. of the total, and makes very little difference to the efficiency of the boiler. Replacing the iron plates by copper of the same thickness merely changes the one per cent. into one-sixth per cent., and does not appreciably increase the efficiency.

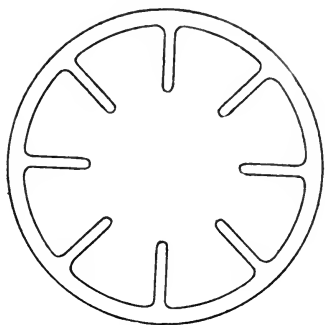
269. Clement attempted to measure the conductivity of a sheet of copper between two and three millimeters thick, by exposing one side to the action of steam, while the other side formed one of the boundaries of a tank of

cold water, the rise in temperature of which was observed. The result gave the conductivity of copper to be  $\cdot 0057$  C. G. S. units, or only one-half per cent. of its true value. Peclet afterwards found that by stirring the water so as to keep up a vigorous circulation, about thirty times as much heat passed through the copper plate; but this was still less than one-sixth of the amount due to the conductivity of the copper. Peclet obtained  $\cdot 178$  C. G. S. units for the conductivity. The explanation of these results is given in the preceding article. In the case of the steam boiler, the principal resistance to the passage of the heat lies in the thin layer of comparatively cool gases which clings to the iron plates and moves forward only at a very slow rate, while the gases near the middle of the flues possess a high velocity, which becomes less and less as the plates are approached, and is practically zero at the surfaces of the plates. The resistance of air is about 20,000 times that of copper, or about 3500 times that of iron. Hence a layer of air  $\cdot 01$  inch in thickness will have resistance seventy times as great as that of an iron plate half an inch thick. Another source of resistance lies in the layer of incrustation, or scale, usually deposited on the side of the boiler plates and tubes in contact with the water, and in a very thin layer of steam formed on the surfaces of the plates when they are free from such incrustation. To diminish the evil effects of this layer of steam, it is important to keep up as vigorous a circulation of the water as possible, so that it may wash the layer of steam, off the plates. Hence, any contrivance which increases the circulation of water in a boiler generally adds to its efficiency.

270. As the resistance to the transfer of the heat is mainly due to the surface layers of furnace gases, or steam, and not to the metal plates themselves, it follows that any arrangement which increases the area of the surface of metal exposed to the furnace gases or to the water, though it may not increase the area of the plates across which the heat has to be conducted, will improve the efficiency of the boiler, unless there is already a sufficient amount of heating surface to cool down the furnace gases

almost to the temperature of the water. The surface exposed to the furnace gases may be called the *collecting surface*, and that exposed to the water the *distributing surface*. The resistance of the layer of furnace gases above referred to will be diminished in proportion as the collecting surface is increased, and the resistance of the layer of scale or

FIG. 38A.



steam will be diminished in proportion as the distributing surface is increased. An arrangement for increasing the collecting surface in tubular boilers has recently been invented in France, and introduced into this country by Messrs. John Brown & Co., of Sheffield. It is known, after the inventor, as the *Serve tube*. Boiler tubes of three inches internal diameter and an eighth of an inch thick are provided with eight longitudinal ribs, radiating inwards for about three-quarters of an inch, and extending the whole length of the tube. The cross section resembles a wheel with eight arms, and the boss and central half of the arms removed. These tubes expose to the furnace gases an area of  $\pi \cdot 3 + 16 \times \frac{3}{4}$ , or 21.42 square inches per inch of length, instead of 9.42 square inches exposed by ordinary three-inch tubes. The collecting area of the tubes is thus more than doubled. The official trials made by the French Admiralty resulted in showing an increased efficiency in the quantity of steam evaporated per pound of coal consumed, of 17 per cent. with natural draught, and of twenty per cent. with forced draught. It is when forced draught is employed, and there is a greatly increased consumption of coal, that the heating surfaces of boilers are most deficient, and it is under these circumstances that the increase in the collecting surface due to the employment of the *Serve tubes* is likely to produce the greatest increase in efficiency. There are some practical difficulties in in-

creasing the distributing surface in tubular boilers in the same way as the collecting surface is increased by the Serve tube. Any ribs or gills attached to the tubes or plates must be so arranged as not to interfere with the free escape of steam from the surface, or with the circulation of the water in the boiler. Subject to these conditions, there is little doubt that an increase in the distributing surface would add considerably to the efficiency of a boiler. It should be noticed that in the case of the Serve tube, the area of the metal across which the heat has to be conducted is not increased, and the heat collected by the projecting ribs has to be conducted through a considerable distance in metal of comparatively small transverse section, so that for this portion of the heat the resistance of the metal is enormously increased; but as it is considerably less than one per cent. of the whole resistance in the case of the thin smooth tubes, there is room for great increase without counterbalancing the advantage gained by the increased collecting surface.

271. Mr. Fletcher, of Warrington, fixes a number of copper pins, about an eighth of an inch in diameter and three-quarters of an inch in length, to the bottom of copper vessels in which water is to be heated by means of gas flames. The pins collect the heat from the flames, and fulfil the same purpose as the ribs in the Serve tubes.

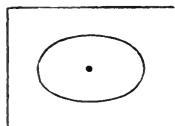
It is sometimes stated that when brass tubes are employed in marine boilers, it is necessary to provide only two-thirds of the amount of heating surface as when iron tubes are employed, on account of the higher conductivity of brass. If brass tubes are more efficient than iron or steel, it is not on account of their greater conductivity, but because they preserve a cleaner surface. The best boiler makers, however, use the same quantity of tubes, whether they are steel or brass.

Some recent experiments have thrown doubt on the conditions under which conduction takes place in iron or steel boiler plates, very anomalous results having been obtained with plates of excessive thickness. To clear up

these doubts it is very important that the actual temperature of the plate should be determined at different points in its thickness, especially at points near to the surface of the plate. This determination is one of extreme difficulty, since it is important that the flow of heat should not be sensibly disturbed by the introduction of the thermometric apparatus. The employment of specially constructed platinum thermometers or thermo-electric junctions buried in the substance of the plate appears to offer the greatest chance of success. The investigation is one of great interest to the boiler maker.

**272. Conduction of Heat in Crystals.** Most substances conduct heat equally well in all directions through their mass, but some crystals and organized structures conduct heat more readily in some directions than in others. Thus, if a plate be cut from a quartz crystal parallel to its axis, covered with paraffin, and heated by a wire at the centre, the paraffin will be melted within an ellipse, whose major axis is parallel to the axis of the crystal, as shown in Fig. 39. This is due to the thermal conductivity of the crystal being greater in the direction of its axis than in any other direction. Paraffin is preferable to wax for this experiment, because it does not soften so much as wax before melting, so that the boundary between the solid and the liquid is much more distinct.

FIG. 39.



If the plate of quartz be cut perpendicularly to the axis of the crystal, the paraffin will melt in a circle, showing that the thermal conductivity is the same in all directions at right angles to the axis of the crystal.

**273. Forbes' Experiments.** In order to compare the thermal conductivities of wrought iron at different temperatures, Principal Forbes employed a bar about 10 feet in length and  $1\frac{1}{4}$  ins. square. One end of this was inserted into a crucible of lead, which was kept melted, great care being taken to preserve the temperature constant throughout the experiment. The rest of the bar was protected from the radiation of the lamp and crucible

by means of a double screen. A number of small holes were drilled at intervals along the upper surface of the bar, and in these holes were placed the bulbs of small thermometers, the holes being filled up with mercury. The thermometers served to determine the form of the temperature curve described in Art. 259. Then, knowing the form of this curve, and therefore the rate of change of temperature at different points along the bar when the flow of heat became *steady*, it was only necessary to determine the amount of heat which flowed across any section of the bar per second, in order to know the conductivity of iron at the temperature of the section. Now, as explained in Art. 257, when the flow of heat is steady, the amount which crosses any section of the bar in each second is equal to the amount radiated by the portion of the bar between that section and the cold extremity. Thus, referring to Fig. 37, Art. 259, the amount of heat passing the section of the bar at *S* is equal to the amount radiated in the same time by the portion *SE* of the bar, since the temperature at every point of the bar itself remains constant. In order to determine the amount radiated, Principal Forbes employed a short length of wrought iron bar of the same section as the experimental bar, and having heated it to the temperature of the hottest part of the experimental bar, he observed the rate at which it cooled by means of a thermometer inserted in a hole drilled in the bar. Knowing the specific heat of iron at different temperatures, the rate at which heat escapes by radiation from each inch of the bar at temperatures varying from that of the room to that of the hottest portion of the experimental bar became known; and hence the amount lost, per second, by any portion of the experimental bar, measured from the cold end, could be determined. This gave the flow of heat across any section, and consequently the absolute conductivity, since the rate of change of temperature was known. The results of this investigation indicated that the thermal diffusivity diminished as the temperature increased, changing steadily from  $\cdot 01337$  foot-minute-degree units at  $0^{\circ}\text{C.}$  to  $\cdot 00801$  units at  $275^{\circ}$ , the unit of heat being taken to be the amount of heat

required to raise the temperature of one cubic foot of iron  $1^{\circ}\text{C}$ . A second experiment with a one-inch bar gave similar results, though in the latter case the difference as well as the conductivity itself was less.

274. By a method similar to that of Principal Forbes, but employing bars of different metals, Despretz determined the relative thermal conductivities of several metals, and Forbes' apparatus is sometimes called after Despretz. In accurate determinations of the conductivity of metals, the measurements should be conducted in vacuo, as air-currents help to accelerate uniform distribution of temperature.

275. Wiedemann and Franz improved upon Forbes' method by employing thin bars, or wires, one end of each of which was heated to a constant temperature, and the variation of temperature along the bar was determined by placing a thermo-electric junction in contact with the surface of the metal. This avoided the errors caused by drilling holes in the bar, as in the experiments of Forbes and Despretz, though the errors thus occasioned were exceedingly small. The relative thermal conductivities of some metals, as determined by Wiedemann and Franz from experiments in vacuo, are given in the following table:—

Silver . . .	100.0	Iron . . . .	10.1
Copper . . .	74.8	Steel . . . .	10.3
Gold . . . .	54.8	Platinum . .	9.4
Brass . . . .	24.0	Lead . . . .	7.9
Tin . . . . .	15.4	Bismuth(in air)	1.8

The conductivity of copper is very greatly diminished by small quantities of impurities.

Tait has repeated Forbes' experiments with great care. He has also found that the thermal conductivity of copper and German silver improves with increase of temperature.

276. It is owing to the high conductivity of copper that a piece of copper gauze may be depressed upon a gas flame without any of the gas which passes through the gauze igniting above it. Similarly, if a piece of fine copper gauze be held over a gas-jet, and the gas ignited

above the gauze, the flame will not be communicated to the gas below. In the gauze-burner the jet of gas mingles with the air as it escapes, and then, passing through a sheet of gauze, is ignited above it. The Davy safety-lamp consists simply of a gauze cylinder, in the interior of which the oil burns, but no flame can be communicated to any inflammable mixture outside the gauze.

**277. Absolute Thermal Conductivities.** The absolute thermal conductivities of a few substances are given in the following table, the units being the centimetre, gramme, and second; that is to say, the unit of conductivity is the conductivity of a substance which transmits the amount of heat required to raise a gramme of water from  $0^{\circ}$  to  $1^{\circ}$  C. across each square centimetre of a plate a centimetre thick, when the temperatures of its faces differ by  $1^{\circ}$  C.

TABLE OF ABSOLUTE THERMAL CONDUCTIVITIES.

Copper . . . . .	·91
Iron . . . . .	·16
Air, Oxygen, Nitrogen, } and Carbonic Oxide }	·000049
Carbonic Anhydride . . . .	·000038
Hydrogen . . . . .	·00034
Water . . . . .	·002
Fir, across fibre . . . . .	·00026
Fir, along fibre . . . . .	·00047
Underground strata (average)	·005.

As stated above, Tait has found that the thermal conductivity of copper improves with increase of temperature. Tait found for the thermal *diffusivities* of

Copper . . .	·076 ( $1 + \cdot0004 t$ )
Iron . . . .	·015 ( $1 - \cdot00144 t$ ),

the units being feet and minutes, and  $t$  denoting the temperature on the centigrade scale. Taking the mass of a cubic foot of copper to be 560 lbs., and its specific heat ·095, and the mass of a cubic foot of iron to be 480 lbs., and its specific heat ·112, Tait's results, translated into C.G.S. units, give for



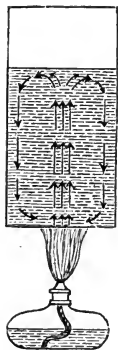
Copper	. . .	·992 (1 + ·0004 t)
Iron	. . .	·196 (1 - ·00144 t).

It must be remembered, in comparing the results of different experimenters, that very slight impurities in metals, or differences in their physical conditions, make very great differences in their thermal conductivities. Tait's result gives 606 units of heat per hour for the amount conducted across a square foot of iron plate three-quarters of an inch in thickness, at a mean temperature of 150° C., with a difference of 1° C. between its faces. This corresponds very closely with the statement in Art. 267.

**278. Convection of Heat.** Pure copper and pure silver are the best conductors of heat known. Animal and vegetable substances are for the most part very bad conductors. Solids are in general very much better conductors of heat than liquids, while the thermal conductivities of gases are extremely small. Both liquids and gases are capable, however, of readily diffusing heat throughout their masses, since the extreme mobility of their parts makes up for their low conducting powers. Heat is therefore generally diffused through liquids and gases by *convection* assisting *conduction*.

**279. Convection Currents.** It is a well-known fact that there is an upward current of air above and around a gas-flame, because the air which has been heated by the flame is less dense than the rest of the air in the room. For a similar reason there is a draught up a chimney when a fire is burning below it. Now, suppose a gas-flame burning in the middle of a room, the doors and windows of which are closed. There will then be an upward current of hot air in the middle of the room above the flame, while a downward current must exist nearer the walls in order to supply the place of the ascending air. In this way the air in the room will be kept in circulation, and heat will be conveyed from the gas-flame to other parts of the room *by convection*. The currents so set up are sometimes called *convection currents*.

FIG. 40.



280. If a little cochineal be thrown into a vessel of water, and heat applied at the bottom, the water will be seen to circulate, an upward current being formed in the middle of the vessel immediately over the flame, while downward currents exist all round the circumference. In this way all the water is brought in turn close to the bottom of the vessel, where it is heated by conduction through the vessel itself; while the heated stream of liquid meeting, during its ascent, with colder liquid, parts with some of its heat to the latter by conduction, and thus by a combination of the processes of convection and conduction the whole of the liquid becomes heated.

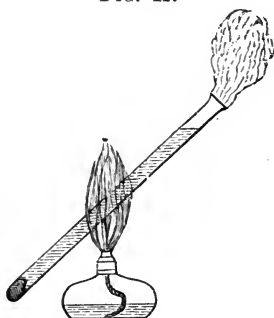
It will be seen from the above that the rate at which a mass of fluid can be heated by a source of heat will depend on the position of the latter relative to it, and will generally be greatest when the heat is applied at the bottom of the fluid.

281. Convection currents may be well shown with the projection lantern by placing a small coil of platinum wire at the bottom of a glass cell which is filled with water. A little cochineal or other colouring matter is placed at the bottom of the cell, and an electric current sent through the platinum wire from copper terminals. The convection currents will then be sufficiently apparent on the screen.

282. When a quantity of fluid is being heated, convection currents serve to carry the hot portions of the fluid away from the source of heat, and to bring the colder portions near to it, and also to mix together the hot and cold portions of the fluid; but the ultimate transfer of the heat to the colder portion of the liquid must take place by conduction. The mixing serves to bring hot and cold portions very near together, and also causes them to present an immense area of surface of contact across which the heat may flow, and thus in two ways accelerates the uniform distribution of the heat, compensating in a great measure for the very low conducting power of most fluids.

283. **Conductivity of Liquids.** The low thermal conductivity of water may be shown by placing a quantity of ice-cold water in a test-tube, and fixing a block of ice at the bottom. On heating the tube at about the middle point of its length, as shown in Fig. 41, the water above this point will be quickly heated by aid of convection currents, and may be made to boil before very much of the ice is melted, on account of the extreme slowness with which heat is conducted downwards to the ice, in which passage it is, after a short time, unaided by convection currents.

FIG. 41.



The extreme slowness with which heat is conducted through water may also be shown by placing a differential thermometer in a tall glass vessel of water, with one bulb about one inch below the surface, and the other as low down as possible. If a tin vessel containing water near its boiling point be held with its bottom just below the surface of the water in the glass vessel, a long time will elapse before the upper bulb of the thermometer becomes sensibly warmed.

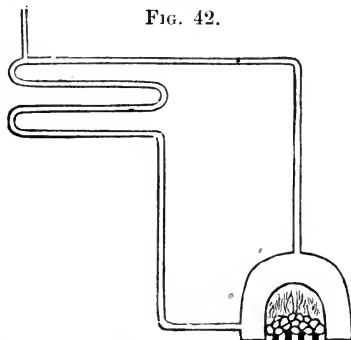
284. Professor Guthrie's apparatus for determining the thermal conductivity of water and other liquids consisted of two cones of brass with platinum bases. Hot water was made to circulate through the upper cone, while the lower cone served as the bulb of an air thermometer. The cones were placed base to base, so that the lower cone had its vertex downwards. In the first instance, the bases of the cones were placed in contact, and the rate at which the temperature rose in the air thermometer was observed. The bases of the cones were then separated through a millimetre, and the space between them filled with a layer of the liquid to be tested. The variation of temperature of the lower thermometer was again observed, and the resistance of the film of liquid to the passage of the heat was deduced from a comparison of the two observations.

It should be noticed that after the introduction of the liquid a sufficient time should be allowed for the liquid itself to become warmed, otherwise the thermal capacity of the liquid will affect the result. As the temperature of the air changes in the lower cone, the temperature of all but the top layer of the liquid film must also change; so that the result of the measurement is never quite free from the influence of the capacity for heat of the liquid.

**285. Heating by Hot Water Circulation.** Let a glass tube be bent so as to form a rectangle, and the ends united so that it may be possible for water to circulate in the tube. At one corner let an opening be made and a small piece of tube sealed into the opening by which water may be poured into the apparatus. Now let the rectangle be supported in a vertical plane, and some coloured water be poured into the lower part of the tube, the rest of the tube being completely filled with uncoloured water. If the flame of a spirit-lamp be applied to one of the vertical sides, the liquid will rise in this side and descend in the other, so that the coloured water will presently reach the top of the tube, then pass along the top branch and descend on the other side. The circulation will continue as long as the flame is applied, and after the colouring matter has become uniformly diffused, the current may be rendered apparent by inserting a few solid particles, which will remain suspended in the water.

**286.** It is by means of a circulation precisely similar to that described in the last Article that houses and other build-

FIG. 42.



ings are generally heated by hot water. The glass tube is replaced by an iron pipe, a coil being inserted in the circuit wherever it is desired to expose a considerable heating surface. The boiler is generally placed in the lowest situation available. To ensure a rapid circulation it is essential that the out-flow pipe for the hot water

should leave the boiler at the highest point, and that the return should enter at the lowest point of the boiler. It is best to carry the outflow pipe vertically upwards as far as possible, so as to have a tall vertical column of water at the highest temperature of the circuit. The heavier water in the cold return pipe overbalances the hot column in the boiler and flow pipe, thus setting up and maintaining the circulation. A small tank is generally inserted at the highest point of the pipes, and connected with a feed cistern by which cold water is supplied to the apparatus when necessary. (Fig. 42.)

287. Suppose that the highest point in the system of pipes is 30 feet above the boiler, and that the temperature of the ascending column may be taken to be  $90^{\circ}\text{C.}$ , and that of the descending column  $60^{\circ}\text{C.}$  The difference in the mean density of the columns is about  $\cdot 022$ . The force urging the circulation is equivalent to a head of water of 8 inches. It is an experimental result in hydraulics that the head of water, balanced by the friction of a length of iron pipe equal to 33 times its diameter, is equal to the head which is capable of producing the observed velocity in the issuing stream, and that every sharp right-angled elbow produces the same resistance. Suppose, then, that the whole system is equivalent to 1,000 feet of three-inch pipe, all in series. Dividing this into lengths, each 33 times the diameter, we obtain 124 lengths. Dividing the head of water by 124, we obtain  $\cdot 0645$  in. head of water, as that to which the velocity of the stream must be due. Now, the velocity due to a fall of  $\cdot 0645$  in. is  $\cdot 588$  feet per second. The velocity of the stream of water is therefore  $\cdot 588$  feet per second, corresponding to a current of 649 gallons per hour; and if the difference in temperature of the water on leaving and entering the boiler is  $45^{\circ}\text{C.}$ , this corresponds to a loss of heat of 292,050 units per hour, which is the amount which the system is capable of communicating to the air of the building. This is equivalent to a steam heating apparatus condensing nearly 500 lbs. of steam per hour, and returning the condensed water to the boiler at a temperature of  $50^{\circ}\text{C.}$ , and it would serve to heat 997,248 cubic feet of air per hour from  $0^{\circ}\text{C.}$  to  $15^{\circ}\text{C.}$

288. Though the quantity of piping has been assumed to be equivalent in resistance to 1,000 feet of three-inch pipe all arranged in series, it is not probable that this would be the actual arrangement. When considerable heating surface is required, the current of water would probably be divided between three or four pipes arranged side by side, in which case the resistance would be nearly proportional to the length of one of the pipes divided by the square of the number of pipes. If a narrower pipe is anywhere employed to carry the whole stream, it must be borne in mind that the resistance of a given length varies approximately inversely as the fifth power of the diameter; so that a single foot of one-inch pipe will introduce as much resistance as 243 feet of three-inch pipe.

289. The amount of heat given out by one square foot of a black metallic surface exposed to air is about 1.91 units per hour for each degree by which its temperature exceeds that of the air, but increases in a higher ratio when the difference of temperature is considerable. Taking the average temperature of the pipes to be  $55^{\circ}$  C. above that of the air, the loss of heat from each square foot of surface would amount to 105 units, provided the pipe were freely exposed to the air all round. In order, therefore, that 292,050 units of heat may be given out per hour, it is necessary that the surface of the pipe should amount to 2,782 square feet, and allowing for the fact that in any ordinary arrangement the whole of the radiating surface is not freely exposed all round, it will be necessary to employ an actual surface exceeding 4,000 square feet, or, say, 6,000 feet of three-inch pipe. Hence the necessity for arranging some of the pipes side by side, in order to secure effective heating.

If the same amount of heat is required, and it is not possible to secure the same rise in the flow pipe, the resistance of the system must be reduced by the employment of larger pipes.

290. Under ordinary circumstances it is desirable that as little water as possible should be drawn from a boiler or pipes employed in heating a building, for every addition of cold water generally brings with it a considerable

amount of lime, salts, or other solid matters in solution, and these are deposited in the boiler and tubes when the water is heated. It is a great mistake to use a hot water heating apparatus as a source of hot water, except where the ordinary water supply is particularly pure.

291. **Maximum Density of Water.** Convection currents were employed by Dr. Joule in order to determine the temperature at which the density of water is a maximum. Two vessels, 4 ft. 6 in. in height, and 6 in. in diameter, were placed side by side and connected near their bases by a tube with a stop-cock, while an open trough served to connect them near their upper edges, a small piece being cut from the side of each. The vessels were filled with water, and a small glass bead made to float in the connecting trough at the top. The temperature of the water in each was adjusted with the stop-cock closed. Then, on opening the cock, the water flowed through the trough, carrying the bead with it towards the vessel in which the density was greatest. If there were no current, the temperature of the vessels being different, one was above and the other below the temperature corresponding to maximum density; and by finding a number of pairs of temperatures, for which there was no convection current, Joule concluded that the temperature corresponding to maximum density was about  $39.1^{\circ}$  F.

## CHAPTER IX.

### ON RADIANT ENERGY.

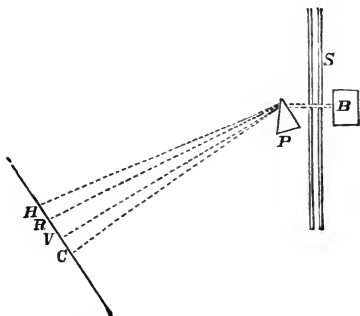
292. If we heat one end of a poker in the fire, but not to redness, and then hold it near the face, a sensation of heat is at once felt, and we infer that some thing or some influence has passed from the poker to the face across the intervening air. If we now replace the poker in the fire and heat it to very dull redness, the same sensation is felt on bringing it to the same distance from the face, but in a stronger degree. If we now hold the poker in front of a black screen, and look at the heated end through a prism, whose edge is parallel to the length of the poker, we see a dull red band slightly wider than the poker itself. If the edge of the prism be towards the left hand, the poker, when viewed through the prism, will appear more to the left than its true position. On heating it still further, till it attains a bright red heat, and again inspecting it through the prism, the red band will appear broader and brighter, and shade off into orange towards the left. On raising the temperature of the poker still more, the red and orange become brighter, while yellow, green, blue, and violet bands, gradually shading into each other, are successively added towards the left. When the violet has made its appearance, the poker will have attained a *white* heat, and no more tints visible to the eye will be added by further increasing the temperature, though the brightness of those already existing will be increased by this means. If the poker, at an intense white heat, be placed at a considerable distance in front of paper prepared with silver salts for receiving photographic images, it will be found capable of blackening it, which will not be the case when the poker is red hot.



The influence proceeding from the poker which produces the sensation of heat on the skin, or of light in the eye, or which is capable of blackening photographic paper, is called the *radiation* from the poker. As such radiation is capable of doing work, it is frequently called *radiant energy*.

**293. The Spectrum.** Now let the following experiment be performed: A mass *B* (Fig. 43) of wrought iron or of platinum is heated to intense whiteness, and placed behind a double screen *S*, in which a narrow slit is cut, and a glass prism is placed in front of the screen with its edge parallel to the length of the slit. A coloured band *RV* will then appear upon a white screen placed in front of the prism, the end *R* towards the edge of the prism being red, and this will be succeeded by orange, yellow, green, blue, and violet towards the end *V*. If now a thermo-electric pile be moved from point to point of the coloured band, or light spectrum, the galvanometer connected with it will indicate that the face turned towards the prism is heated, and this will be the case even beyond the end *R*. If in like manner photographic paper prepared with silver salts be placed at different parts of the spectrum, it will be blackened only in the blue and violet spaces, and in the space *VC* beyond the extreme violet. Let these observations be continued while the mass of metal cools. We observe, first of all, that photographic paper made to pass across the spectrum between *V* and *C* is gradually less and less blackened, till at last it is quite unaffected, while the intensity of the light spectrum gradually diminishes. The light at the extreme end of the violet then vanishes completely, and presently the whole of the violet is gone, the blue follows, then the green, and so on; while the galvanometer registers a

FIG. 43.



gradual diminution in intensity in the radiation at the red end of the spectrum and in the parts beyond the red. Before all the light spectrum has vanished let the glass prism be replaced by a prism of rock salt. The galvanometer will immediately indicate a great increase in the intensity of the radiation falling on the thermo-electric pile placed beyond the red end of the spectrum, and will also indicate the presence of rays at much greater distances beyond the red than were indicated when the glass prism was used. As the metal *B* continues to cool, the green, yellow, and orange lights vanish in succession; and at last, when the metal is reduced below a red heat, the last trace of red vanishes. The galvanometer, however, continues to indicate the presence of radiation beyond the red end of the spectrum, though it now gives no indication of heat when the thermo-electric pile is placed in what *was* the illuminated portion of the spectrum, showing that when the light vanished the rays capable of heating the junctions in the thermo-electric pile vanished with it. As the mass of metal becomes cooler we have to move the thermo-electric pile still farther from the red end of the original spectrum in order to get indications of radiation, and at length the metal becomes too cold to produce any sensible effect on the pile wherever placed.

This experiment may be modified, so as to be more easily performed, by substituting an electric light for the white-hot metal, and an oil-lamp for the metal when at a bright yellow heat, a piece of metal being conveniently employed for lower temperatures; but this interferes with the continuity of the results.\*

294. In the solar spectrum the greatest intensity of the radiation, as measured by the thermo-electric pile, is a little beyond the extreme red, while the greatest amount

\* Instead of using a double screen, as shown in the figure, it is much better to employ a thin screen, and to place a lens of the same material as the prism between the latter and the slit, in such a position that the slit is in its principal focus, a second lens of the same material being placed very close to the prism on the other side of such focal length that the second screen passes through its principal focus.

of radiation capable of affecting photographic paper prepared with silver chloride, is just beyond the extreme violet end of the luminous spectrum.

Different silver salts are most sensitive to rays in different parts of the spectrum, so that the rays which act most powerfully on silver bromide are not identical with those which act most powerfully on the chloride or iodide. Other metallic salts again are sensitive to rays in very different parts of the spectrum, certain iron salts being most sensitive to rays which lie beyond the extreme red ; so that it is only for particular compounds that the violet, or ultra-violet rays are the most "actinic." For other compounds other rays are "actinic," and the distinction of heating, luminous, and actinic rays is entirely without foundation except in the case of luminous rays, and here the distinction is subjective rather than objective, and depends on the particular eye.

295. From the experiments just described, we learn that a solid body, when heated below redness, emits rays capable only of producing heat in bodies on which they fall, while at higher temperatures it emits rays capable of producing the sensations of red, orange, yellow, green, blue, and violet light, while the intensity of the heat-producing rays is increased. At still higher temperatures it also emits rays capable of producing photographic pictures with silver salts, and of developing heat, but not causing the sensation of light.

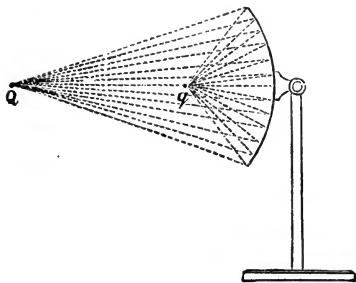
From this experiment we also infer that the *same* rays which produce the sensation of light in the eye, produce heat when allowed to fall upon other bodies, and hence we can make no distinction between light and what has been erroneously called radiant heat. The same is true of the ultra-violet radiation, though the energy of these rays is generally too feeble to produce much current in the thermo-electric pile.

296. **Nature of Radiant Energy.** If it be true that *some* of the rays which produce the sensation of heat are *identical* with those of light, since we cannot suppose that these rays essentially differ in kind from the other

rays which produce the sensation of heat, but not that of light, we must conclude that the *whole* of the radiation which produces heat in bodies absorbing it is of the *same nature* as light, and therefore consists of vibrations of the luminiferous ether, and from the distribution of these rays over the luminous and non-luminous portions of the spectrum, we are led to infer that the dark rays differ from one another, and from the luminous rays, in exactly the same way as the latter differ among themselves, that is, in wave-length; and that as the wave-lengths of the rays continue to increase as we pass along the spectrum from violet to red, so this increase continues beyond the red end. Hence, bodies at a low temperature give out only those rays whose wave-lengths are very long compared with the length of a wave of light of any particular colour. Of the identity of the nature of calorific radiation and light we shall meet with further evidence shortly.

**297. Laws of Reflection.** Again, if it be true that all the rays which are capable of producing heat are of the same nature as light, we should expect that they would be subject to the same laws of reflection and refraction. In the experiments just described, we have demonstrated the refraction of the dark rays by a prism. The reflection

Fig. 44.



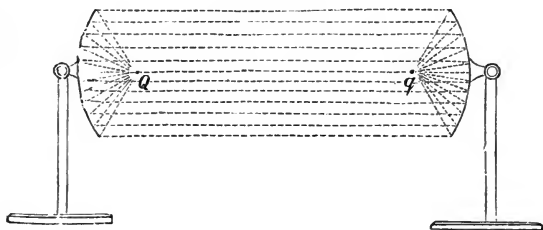
of these rays may be shown thus:—Take a concave mirror *AB* (Fig. 44), and place at *Q* a small source of light; then the point *q*, at which the image of *Q* is formed by the mirror, can be readily determined. Now replace the luminous body at *Q* by a piece of metal heated just below redness, and place at *q* a delicate

thermometer with a blackened bulb, or a thermo-electric junction connected with a galvanometer. The latter will immediately indicate a high temperature; but if it be moved from *q* to any other point at the same distance

from  $Q$ , no such effect will be produced, showing that the dark radiation from  $Q$ , which falls on the mirror, is brought to a focus at  $q$ , and that therefore these rays are subject to the same laws of reflection as those of light.

298. If two parabolic mirrors be placed opposite to one another, so as to have the same axis, as in Fig. 45, and the hot body be placed at  $Q$ , the focus of one mirror, while the thermometer is placed at  $q$ , the focus of the second, the distance between the mirrors may be very greatly increased without much diminishing the indication of heat by the thermometer. This follows from the fact that the rays pass from one mirror to the other without sensibly heating the air; and, being all nearly parallel in their passage (if the body at  $q$  be small), nearly all the rays leaving the first mirror must fall on the second.

FIG. 45.



If a luminous body, such as an oxy-hydrogen light, be placed at  $Q$ , and a quantity of finely powdered chalk or lime scattered through the air, the course of the light between the mirrors, and its accumulation in the neighbourhood of  $q$ , will be distinctly visible. The same applies in the case of the lens mentioned below (Art. 306).

299. The preceding experiments prove that the laws of reflection for other rays are the same as those for light, viz. :—

(1) *The incident and reflected rays are in one plane with the normal to the reflecting surface at the point of incidence, and on opposite sides of it.*

(2) *The incident and reflected rays make equal angles*

*with the normal to the reflecting surface at the point of incidence.*

**300. Relation between Emission and Absorption.** In the experiment described in Art. 293, we found that on substituting rock-salt prism for the glass prism, the galvanometer indicated an increase in the radiation falling on the pile when placed beyond the red end of the spectrum. If similar pieces of polished glass and rock-salt be exposed to the same radiation, especially if the greater part of it consist of dark rays (as is usually the case), the glass will become more highly heated than the rock-salt. From these two observations we infer that *glass absorbs much of the dark radiation which rock-salt transmits*, the glass being thereby heated. This is equivalent to saying that rock-salt is more *diathermanous* than glass, although its power of transmitting *luminous* rays, or its *transparency*, is no greater than that of glass. All bodies absorb some of the rays incident upon them, that is, are opaque (or adiathermanous) to rays of certain wave lengths. Metals are almost perfectly opaque to all rays. (Gold in *exceedingly thin* leaves transmits green light, and silver blue light.) *All bodies absorb the same kind of rays which they themselves emit when heated.* This accounts for the dark lines in the solar spectrum, and is the basis of solar and stellar chemistry. Thus sodium vapour absorbs the yellow rays corresponding to the double D line of the solar spectrum, and emits only these luminous rays when incandescent, so that it is impossible to make sodium vapour white-hot.

**301.** When a gas or vapour emits the same *kind* of light at very high temperatures and at comparatively low ones, it is possible to *reverse* its spectrum by causing the light from the electric arc to pass through a Bunsen flame containing the substance in question. Black lines will then appear corresponding exactly to the bright lines which are visible when the Bunsen flame is removed and a small quantity of the substance placed in the electric arc. (The lines of course are not absolutely black, being illuminated by the light from the gas flame, but they appear black by contrast.) If the substance in the electric arc produces

bright lines, which are not visible in the flame spectrum, these lines, of course, cannot be reversed by this method. The reversal of the bright lines in the spectra of metals, or other substances, is an example of the "Theory of Exchanges."

302. The property of *selective absorption* is possessed in some degree by all known bodies, however transparent. The clearest glass absorbs not only the non-luminous radiation beyond the visible red of the spectrum, but also absorbs nearly all the chemically active radiation which is more refrangible than the violet. In fact, it transmits very little radiation beyond that which is sensible to the eye. Many bodies which appear to be much less transparent than glass transmit more of the *total* radiation from the sun or the electric light. As above stated, rock-salt transmits nearly the whole of the ultra-red radiation; quartz, on the other hand, transmits rays lying very much beyond the violet, so that the spectrum of the electric light produced by quartz prisms and lenses can be traced to a distance beyond the extreme violet, equal to six or seven times the length of the visible spectrum. These rays may be rendered apparent either by their photographic effects, or by causing them to fall upon certain *fluorescent* substances, like sulphate of quinine, æsculine, etc., which absorb them and then shine with a visible light.

303. The fact that glass absorbs so much of the non-luminous radiation which falls upon it, accounts for its efficiency as a fire screen as well as its action in hot-houses. By far the greater part of the radiation from an ordinary fire is non-luminous, and this is absorbed by a glass fire-screen, rendering the screen itself warm, and then being radiated by the glass in all directions, so that the portion reaching any person in the room is much less than it would be if the screen were not present, while the cheerful luminous radiation is nearly all transmitted. When glass is employed in frames and hot-houses, a very large percentage of the solar radiation is transmitted, since very much of it is luminous, but of the non-luminous radiation from the plants, earth, and other objects below

the glass, nearly all is absorbed by the glass which returns through its own radiation a great portion of that which it absorbs to the objects beneath it. The glass thus acts like a trap or ratchet, allowing the radiation to pass in one direction, but refusing to allow it to return.

Air which contains aqueous vapour acts like glass, absorbing very much of the non-luminous radiation from the earth, etc., while it transmits the radiation of shorter wave-lengths from the sun. Air devoid of aqueous vapour is much more diathermanous to non-luminous radiation than air containing it. The same property of absorbing non-luminous radiation is possessed by olefiant gas in a very marked manner.

**304. Diathermancy.** The following table shows the percentage of the total radiation from the four sources mentioned, which is transmitted by a plate of the several substances named, one-tenth of an inch in thickness. It will be noticed that rock-salt transmits with equal facility both luminous and non-luminous radiation while Iceland spar, glass, tourmaline, selenite, alum and ice, are almost opaque to radiation from a source at a low temperature. It is to Melloni we are indebted for these results.

	NAME OF SOURCE.			
	Locatelli Lamp	Incandescent Platinum.	Copper at 400° C.	Copper at 100° C.
Rock-salt . . .	92·3	. 92·3 . .	92·3 . .	92·3
Fluor spar . . .	72	. 69 . .	42 . .	33
Iceland spar . . .	39	. 28 . .	6 . .	0
Glass . . . .	39	. 24 . .	6 . .	0
Quartz . . . .	38	. 28 . .	6 . .	3
Tourmaline (green)	18	. 16 . .	3 . .	0
Selenite. . . .	14	. 5 . .	0 . .	0
Alum . . . .	9	. 2 . .	0 . .	0
Ice . . . .	6	. 0·5 . .	0 . .	0

**305. Absorption by Gases.** The following table gives the relative absorbing powers of a few gases at a pressure of one inch of mercury, the length of the tube containing the gas being 2 ft. 8 in., and the source of radiation a plate of copper heated by a Bunsen flame. These results were obtained by Professor Tyndall.



	Absorption.
Air . . . . .	1
Oxygen . . . . .	1
Nitrogen . . . . .	1
Hydrogen . . . . .	1
Chlorine . . . . .	60
Carbonic oxide . . . . .	750
Nitrous oxide . . . . .	1860
Sulphuretted hydrogen .	2100
Ammonia . . . . .	7260
Olefiant gas . . . . .	7940
Sulphurous anhydride .	8800

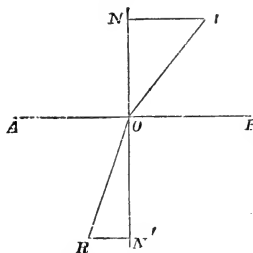
306. **Absorption by Solution of Iodine.** A solution of iodine in bisulphide of carbon absorbs the *whole* of the luminous radiation from any source, while it transmits a large portion of the dark radiation. If a beam of solar light be brought to a focus in the air by a large achromatic lens, and a wide vessel containing this solution be introduced into its path, the *whole* of the *light* will be cut off. (This Professor Tyndall proved by placing his eye so that the focus fell on the retina, defending all but the pupil by a perforated screen.) If now a small piece of platinum be placed at the point which was occupied by the luminous focus, the platinum will be heated to whiteness, showing that the dark rays are brought to a focus at the same point as the light by the achromatic lens. (If a common lens be used, the best focus of these rays is a little beyond that of the red rays.)

307. **Laws of Refraction.** This experiment proves that dark radiation is subject to the same laws of *refraction* as light, viz.:—

(1) *The incident and refracted rays lie in one plane with the normal to the refracting surface at the point of incidence and on opposite sides of it.*

(2) *In passing from one given medium to another, the sine of the angle of incidence bears to the sine of the angle of refraction a constant ratio, which depends only on the character of the radiation.*

FIG. 46.



Thus, if  $IO$  represent a ray incident at  $O$  from air on a surface  $AB$  of glass, and  $OR$  the refracted ray,  $NN'$  being normal to  $AB$ ,  $ION$  is the angle of incidence and  $RON'$  the angle of refraction; and if we make  $OR$  equal to  $OI$ , and draw  $IN$ ,  $RN'$  perpendicular to  $NN'$ , the ratio of  $IN$  to  $RN'$  is independent of the angle of incidence, and is called the index of refraction of the particular ray between air and glass.

**308. Selective Absorption.** It is worthy of mention that when radiation passes through a considerable thickness of any particular medium, the latter absorbs all the rays which it is capable of absorbing, and the radiation may then be made to pass through another portion of the medium without being sensibly absorbed by it. Thus, if a beam of solar light be made to pass through a few inches of ice, it will melt the latter in the interior in the form of beautiful hexagonal flowers, like snowflakes, whose planes coincide with the planes of freezing of the ice, but it may afterwards be made to pass through a lens of ice without damage to the latter, and may thus be brought to a focus in which platinum may be heated to redness.

**309. Polarisation.** It is not only in reflection and ordinary refraction that non-luminous radiation obeys the same laws as light. It can also be doubly refracted by crystals and polarised. If light be allowed to fall upon a Nicol's prism, or a plate of tourmaline, all the vibrations of the transmitted light are reduced to one plane, the rest being absorbed by the tourmaline or totally reflected within the Nicol. If a second Nicol or plate of tourmaline be placed in the course of the light in one position, the whole of the light will be transmitted; but if the second Nicol or tourmaline be turned at right angles to this, the whole will be cut off. If a thermo-pile be placed in the course of the light, it will be found that when the light is transmitted the pile is affected, but when the

light is cut off no current is produced by the pile ; showing that all calorific radiation is cut off with the light. Hence the non-luminous radiation must be polarised by the first Nicol or tourmaline, and, in the case of the Nicol, this shows that it is also subject, like light, to double refraction.

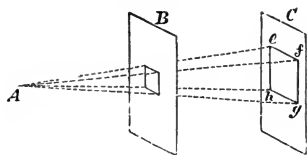
**310. Light and Non-luminous Radiation.** We see, then, that non-luminous radiation accompanies light in reflection, refraction, double refraction, and polarisation, and is subject to precisely the same laws as the light itself. Hence we infer that it is of the same nature as light, which is generally supposed to consist of a vibratory motion of the luminiferous ether, a very rare, jelly-like solid, which is believed to pervade the whole visible universe. By exploring the spectrum with the thermo-pile, we learn (Art. 293) that in most cases the greater part of the radiation is less refrangible than red light, and the vibrations constituting it are therefore longer and less rapid than those of red light. Fluorescent bodies indicate that when the source is at a white heat, there is also emitted radiation more refrangible, and therefore of shorter wave-length, than violet light ; but this generally has very little heating effect. We conclude that *all* the radiation from hot bodies is of the same nature as light, but only that portion of the radiation whose period of vibration lies between certain limits produces the sensation of light. These limits vary somewhat with different human eyes, and there is no reason apparent why some animals should not possess eyes capable of being affected by the radiation from bodies far below a red heat, or even at ordinary temperatures, so as to see everything clearly in a room perfectly dark to us, provided the temperature is above a certain point.

Light and non-luminous radiation being thus identical in nature, and obeying the same laws, all the propositions of optics are equally true for all heat-producing rays.

**311. Law of Inverse Squares.** Through uniform media radiation is propagated in straight lines.

Suppose radiation emitted from a very small body at *A* to pass through a rectangular hole in a fixed screen *B*,

FIG. 47.



and to fall on a second screen  $C$ , which is placed parallel to  $B$ . The whole of the radiation which passes through the hole in  $B$  will fall on a rectangular area on  $C$ , and the area of this rectangle will be proportional to the square of the distance of the screen  $C$  from  $A$ . But wherever the screen  $C$  is placed, the amount of radiation which reaches it is the same, viz.: that which passes through the hole in  $B$ . Therefore, the amount of radiation which falls on each unit of area of the rectangle  $efgh$  varies inversely as the area of this rectangle; that is, inversely as the square of the distance of  $C$  from  $A$ . Hence, *the intensity of the radiation at any point varies inversely as the square of the distance of the point from the source.* This is known as the law of inverse squares.

312. If the direction of the screen  $C$  be changed so that it receives the radiation from  $A$  very obliquely, it will be seen that the area of  $C$  on which this radiation which passes through the hole in  $B$  falls, will be increased, and the amount of radiation falling on the unit of area will be correspondingly diminished. In this way it will be seen by the student who has read Trigonometry that *the intensity of the radiation received by  $C$  is proportional to the cosine of the angle of incidence.*

313. **Diathermancy of Air.** We have said (Art. 254) that when heat is transmitted by radiation, it may travel from one point to another *without heating the medium through which it passes.* That radiation will pass through air without sensibly heating it, may be shown by the following experiment:—Let a small piece of platinum be covered with lampblack and placed in the bulb of a differential thermometer. By means of a lens let some of the solar radiation be brought to a focus in the air within the bulb, care being taken that none of the rays fall upon the platinum. The liquid in the tube will give a very slight indication of heat (and this slight indication arises principally from the heating of the glass of the bulb and consequent heating of the air in contact with

it), showing that the air is not sensibly heated by the rays. Now let the lens be moved till the focus falls upon the platinum. The latter will be almost immediately heated to redness, and will then heat the air around it, and the liquid in the tube will be rapidly driven towards the other bulb.

**314. Radiation and Absorption.** Different surfaces possess different powers of radiation; this may be shown by constructing a metal cube, polishing one side, leaving a second with a dead metallic surface, covering a third side with velvet or other material, and a fourth with lampblack. If the cube be filled with boiling water, and its sides successively presented to the thermo-electric pile at a distance of, say, one foot, the galvanometer will show that the radiation is greatest from the side covered with lampblack, next greatest from the velvet side, and least from the polished side. This cube is called a Leslie's cube. By covering its sides with different materials, we may determine their relative radiating powers.

Similarly, if two equal tin canisters be filled with hot water, and one expose its polished surface to the air, while the other is closely covered with flannel, the latter will cool more quickly than the former, on account of the radiating power of the flannel. If the flannel do not *touch* the canister, it will serve to keep in its heat.

From these experiments we conclude that *surfaces which reflect well radiate badly, while bad reflectors are good radiators.*

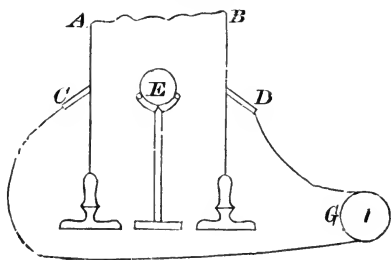
Again, if the bulb of a thermometer, or face of a thermo-electric pile, be covered with lampblack, it will absorb radiation incident upon it much more readily than if it be kept clean. A quantity of water can be boiled in a kettle whose exterior has been blackened, much more readily than in a similar kettle kept bright. Combining these results with those of the experiments just described, we infer that *surfaces which reflect well both radiate and absorb badly, while surfaces which are bad reflectors are good radiators and also absorb a large portion of the radiation incident upon them.*

We may mention that the absorbing power of any sur-

face is generally different for the radiation from different sources. Lampblack absorbs nearly all the radiation incident upon it from any source, and for this reason the bulb of the thermometer, or face of the thermoelectric pile, employed to measure radiation, should be covered with lampblack. The radiation emitted by any body always increases as the temperature of the body is raised.

315. The difference between the absorbing power of a bright metallic surface and of a blackened surface can be

FIG. 48.



conspicuously shown by the apparatus sketched in Fig. 48, in which *A* and *B* represent two discs of tinned iron, to the backs of which are soldered small bars of bismuth, *C* and *D*. The disc *A* is bright, but *B* is covered with lampblack. The discs are united by a copper wire *AB*, and the

bismuth bars are connected with the electrodes of a galvanometer *G*. A red-hot iron ball *E* is placed midway between the discs. Now, if the junction of the bismuth bar *D*, with the iron plate *B*, only be heated, an electric current will pass from the bismuth to the iron across the hot junction, and so go through the galvanometer from *C* to *D*. If the junction of *A* and *C* alone be heated, the current will go through the galvanometer in the opposite direction; while if both junctions be heated equally, no current will be produced. After introducing the hot ball between the plates, it is found that the current passes through the galvanometer from *C* to *D*, indicating that the junction of the bismuth with the blackened plate *B* is more heated than the other, whence we infer that the plate *B* absorbs more radiation than the bright plate *A*. By removing *B* farther from the source of heat, we can find a position for *B* in which no current passes, and then, comparing the distance of *A* and *B* from *E*, can calculate

approximately the relative absorbing powers of the two surfaces. In one experiment, when the hot ball was midway between the plates, the solder uniting the bismuth bar to the blackened disc was melted, and the bar dropped off, while that on the bright disc remained perfectly sound.

316. If some letters be written in ink on a piece of platinum foil, on heating the foil to redness a little oxide of iron is left clinging to the platinum. If the foil be now heated in a Bunsen flame in a dark room, the letters will appear much brighter than the rest of the foil, showing that the iron oxide, which reflects much less light than the platinum, radiates much more when incandescent. Now let the platinum be turned over in the flame so that the plain side may be seen. The letters will still appear legible (though inverted), but in this case the letters appear dark on a bright ground. The reason of this is that more radiation being emitted from the iron oxide than from the platinum, the metal is cooled where the letters are written more than elsewhere, and the reverse side presenting everywhere a uniform surface of platinum, the amount of radiation emitted depends only on the temperature of the metal, and is therefore less behind the letters than elsewhere.

317. **Rate of Cooling.**—We have stated (Art. 314) that the radiation emitted by a hot body increases as the temperature rises. The amount which it *receives* depends, of course, upon the temperature of surrounding bodies, and if it be placed in an enclosure of uniform temperature below that of the body, the rate at which it will lose heat will depend on this difference. Newton came to the conclusion that the rate of cooling is proportional to the excess of the temperature of the body over that of the enclosure. This result is known as Newton's law of cooling, but is not strictly true. Dulong and Petit investigated the rate of cooling, both in air and in vacuo, of a thermometer bulb which was placed in the interior of a thin copper sphere, the sphere being immersed in water at a known temperature. Their results indicate that the rate of cooling depends upon the absolute temperature

both of the body and the receiver, and not simply upon their difference. For the same difference of temperature, the rate of cooling increases with the temperature of the enclosure. Thus, the difference of temperature being  $100^{\circ}$ , the rate of cooling when the enclosure was at  $80^{\circ}$  C., and the thermometer at  $180^{\circ}$  C., was 1.87 times the rate when the enclosure was at  $0^{\circ}$ , and the thermometer at  $100^{\circ}$  C.

318. Dulong and Petit found that the rate of cooling of a body in vacuo in an enclosure at uniform temperature might be represented by the formula—

$$m (a^{t+\theta} - a^{\theta}),$$

where  $\theta$  represents the temperature of the enclosure, and  $t$  the excess of the temperature of the body over that of the enclosure. They also found the value of  $a$  to be 1.0077. The value of  $m$  depends on the body and enclosure concerned.

This formula suggests the idea that the loss of heat of the body to the enclosure is represented by  $m (a^{t+\theta} - 1)$ , and its gain from the enclosure by  $m (a^{\theta} - 1)$ .

319. Stefan, making use of Dulong and Petit's results, concluded that the radiating power of a body varies directly as the fourth power of its absolute temperature, and showed that this law agreed more accurately with the figures obtained by Dulong and Petit than did the law just given, which they deduced from their own work. Clausius afterwards obtained the same law of radiation from purely theoretical considerations, and this law is now more generally accepted than that of Dulong and Petit.

320. Not much is known of the actual amount of heat radiated from different surfaces under given conditions. Forbes determined the loss of heat by radiation from his iron bars at different temperatures in the course of his investigations on the thermal conductivity of iron. Nicol obtained the following figures for the loss of heat in one minute from a square foot of bright and of black copper in a blackened enclosure, the pressure of the air in the enclosure being represented by 30, 4, and .4 inches of



mercury in the successive experiments, and the unit of heat being that required to raise 1 lb. of water  $1^{\circ}$  C., the temperature of the copper being  $58^{\circ}$  C., and that of the enclosure  $8^{\circ}$  C.

	30 ins.	4 ins.	4 in.
Bright Copper . .	1.09 .	.51 .	.42
Black Copper . .	2.03 .	1.46 .	1.35

From these figures it will be seen how important a part is played by the air in promoting loss of heat from a hot body. The same action is sometimes forcibly impressed upon us if we take hold of an electric incandescent lamp which has been badly exhausted. It is easy to grasp in the hand a well-made 16-candle lamp when burning at full power; but if the lamp has been imperfectly exhausted, it is far too hot to hold. A boiler exposing 300 square feet of uncovered iron, and containing steam at a pressure of 100 lbs. per square inch, will probably lose about 100,000 units of heat per hour, corresponding to an extra consumption of about twenty pounds of coal.

### THEORY OF EXCHANGES.

321. If in the focus of one of the conjugate mirrors described in Art. 297 we place the bulb of a thermometer, while a block of ice is placed in the focus of the other, the thermometer will indicate a decrease of temperature. The same would happen if the thermometer were placed in a cavity in a block of ice without touching the latter. The reason of this is that the thermometer *emits more radiation to the ice than it receives from it*. If the thermometer be placed in a room whose walls, as well as everything in the room, are at the same temperature as the thermometer, the latter will indicate no change; while if the thermometer be originally at a lower temperature than the room, its temperature will increase; if, on the other hand, it be at a higher temperature than that of the room, its temperature will decrease. The behaviour of the thermometer in all these cases is completely explained by Prevost's *Theory of Exchanges*.

This theory asserts that every body radiates at a rate depending only on the nature and temperature of its

surface, while it absorbs a certain fraction of the radiation it receives from other bodies, which fraction is a measure of the absorbing power of its surface for the particular kind of radiation which it receives.

322. From the fact that a surface appears equally bright in whatever direction it is viewed, it follows that

*The amount of radiation emitted by any surface is greatest in the direction of the normal to the surface, and that for other directions the amount is proportional to the cosine of the angle which the direction makes with the normal.*

Also we have seen (Art. 311) that

*The intensity of radiation at any point varies inversely as the square of its distance from the source.*

From these data it may be shown geometrically that if two bodies, *A* and *B*, which emit the same amount of radiation per unit of area of their surfaces, be placed near to each other, the amount of radiation reaching *A* from *B* is equal to the amount from *A* which falls on *B*.

323. Suppose a thermometer to be placed with its bulb at the centre of a hollow sphere, which is initially at the same temperature as the thermometer, and any part of whose surface is capable of radiating ten times as much as an equal portion of the surface of the thermometer bulb at the same temperature. Experiment shows that the thermometer always retains the temperature of the enclosure in which it is placed. Suppose also that the surface of the sphere absorbs one-half of the radiation which falls upon it from the thermometer, the other half being returned to the thermometer by reflection. Then, if the surface of the sphere radiated as much per unit of its area as that of the thermometer, just as much radiation would reach the thermometer from the sphere as the thermometer itself gives out; for, while the whole of the latter reaches the sphere, only a very small part of the radiation from the surface of the sphere reaches the small bulb at its centre, viz., that part of the radiation which leaves the surface of the sphere along the radius, or nearly in that direction. But since the sphere radiates ten times as much per unit of area as does the thermo-

meter bulb, it follows that ten times as much radiation reaches the thermometer from the sphere as reaches the sphere from the thermometer; and since the thermometer retains the same temperature, and no heat can be lost by it, except by entering the sphere, it follows that in each second *the thermometer must absorb the same amount of heat from the sphere as the sphere absorbs from the thermometer.* But the sphere absorbs half of the total radiation from the thermometer. Therefore, the thermometer must absorb one-twentieth of the radiation which reaches it from the sphere. Hence, while the spherical surface absorbs one-half of the total radiation from the thermometer, the latter absorbs but one-twentieth of that which it receives from the sphere, or the absorbing powers of the two surfaces are directly proportional to their radiating powers. Now, whatever be the nature of the surfaces of the sphere and thermometer, the latter will always retain the temperature of the former; hence we conclude, that if we measure the absorbing power of a surface by the ratio of the amount of radiation it absorbs to the whole amount incident upon it,

*The absorbing power of any surface, A, for radiation emitted from any other surface, B, is to the absorbing power of B for radiation reaching it from A, as the radiating power of A is to that of B.*

This relation is true not only of the whole radiation from the surface, but also of any selected constituent thereof.

Balfour Stewart has shown that the radiation from a thick plate of rock-salt is greater than that from a thin plate at the same temperature, which indicates that in transparent bodies radiation takes place within the substance of the body itself.

324. As we have stated above, it may be shown by geometry that if two bodies, *A* and *B*, which emit the same amount of radiation per unit of area of their surfaces, be placed near each other, the amount of radiation from *B* which falls on *A* is equal to the amount from *A* which reaches *B*. Hence, if any number of bodies, all at the same temperature, be placed in the neighbourhood

of one another, each will emit radiation to, and receive radiation from, all the others; and the amount of radiation from any body  $B$  which any other body  $A$  absorbs, is exactly equal to the amount of radiation from  $A$  absorbed by  $B$ .

Now, suppose that two bodies  $A$  and  $B$  are near to each other, and the temperature of  $B$  is higher than that of  $A$ ; also suppose that the surface of  $B$  has ten times the radiating power possessed by  $A$ , and therefore ten times the absorbing power; and moreover that  $B$  absorbs half the radiation falling upon it from  $A$ . Then, by what we have proved,  $A$  will absorb one-twentieth of the radiation reaching it from  $B$ . But since  $B$  is at a higher temperature than  $A$ , *more than* ten times as much radiation reaches  $A$  from  $B$  (of which  $A$  absorbs one-twentieth) as reaches  $B$  from  $A$  (of which  $B$  absorbs one-half). Hence,  $A$  absorbs more radiation from  $B$  than  $B$  absorbs from  $A$ , and thus the temperature of  $A$  rises, while that of  $B$  falls. Thus, if  $A$  be a block of ice, and  $B$  a thermometer originally at  $15^{\circ}\text{C}$ ., and everything else in the room, including the walls, be at  $15^{\circ}\text{C}$ ., the temperature of the thermometer will fall on account of the presence of the ice; while if the ice be removed the thermometer is capable of receiving radiation from the wall, in the directions previously subtended by the ice, and thus its temperature will be again raised to  $15^{\circ}\text{C}$ ., and kept there.

325. The following is a brief summary of our results.

Prevost's Theory of Exchanges may be thus stated:

*Every body emits radiation in all directions at a rate depending only on its temperature and the nature of its surface, while it absorbs radiation from all bodies to which it is exposed.*

Combining this with the known fact that a body placed in an enclosure kept at constant temperature acquires the temperature of the enclosure, we see that the following law must be true, viz:—

*The absorbing power of any surface,  $A$ , for radiation emitted from any other surface,  $B$ , is to the absorbing power of  $B$  for the radiation from  $A$  as the radiating power of*

*A is to that of B; the absorbing power of a surface being measured by the ratio of the radiation it absorbs to the whole radiation incident upon it.*

The absorbing power of any surface for rays of different wave-lengths is proportional to its radiating power for the same rays.

326. When radiation falls upon a transparent, or diathermanous body, as rock-salt or air, part of the radiation is absorbed within the body, and the absorption goes on as the radiation traverses it until all those particular rays which it is capable of absorbing have been filtered out, and the remaining radiation is then transmitted by the body without sensible diminution (Art. 308). On examining the spectrum of radiation after traversing such a medium, it will be found to be crossed by a number of absorption bands, the radiation corresponding to which has been removed, leaving black spaces in the luminous portion of the spectrum, spaces in which a thermo-electric junction is unaffected in the parts beyond the red, and spaces in which photographic paper is unblackened beyond the violet. If the body be heated so as to become self-luminous, it will be found that it emits those rays which in the previous case it absorbed.

327. Not only is it true that bodies (including gases) absorb radiation of the same wave-length as they themselves emit, but if a body absorb radiation which is polarised in one particular plane, the radiation it emits when self-luminous will be found polarised in the same plane. Thus a crystal of tourmaline absorbs all vibrations perpendicular to its axis, transmitting only those parallel to its axis. If such a crystal be heated to redness, and the radiation examined through another crystal of tourmaline, or a Nicol's prism, the vibrations will be found all to take place in the plane perpendicular to the axis of the crystal.

328. **Nature of Flame.** Flame consists of gases heated so as to become self-luminous. Now, gases (as we have seen in the case of air in Art. 313) possess very little power of absorbing radiation, and consequently

possess very little power of emitting it. Hence a flame which contains within it no solid particles generally emits very little light. The flame of hydrogen burning in air or pure oxygen is scarcely visible in ordinary daylight, but if a piece of lime be heated by it we obtain the oxy-hydrogen lime-light. When the pressure under which gases are burning is very much increased, and the density of the gases consequently increased, they give out much more light than under ordinary circumstances.

329. The radiation from an ordinary gas or candle flame is due principally to the presence of solid particles of carbon within it, and the illuminating power of coal gas depends on the amount of carbon it contains. An ordinary candle, or coal-gas flame, consists of three distinct portions. The central portion, *A*, Fig. 49, called the area of non-combustion, contains gas which has not yet

FIG. 49.



met with air, and is therefore not burning, and being at a comparatively low temperature is non-luminous. Surrounding this area is another, in which the gas has met with a partial but insufficient supply of oxygen, and in it a portion of the gas is burned, while solid particles of carbon formed by its decomposition are raised to a very intense heat and emit much radiation. This is called the area of partial combustion, and is marked *B* in the figure. Beyond this is the area of complete combustion, *C*, in which the particles of carbon are burned up; and since this area consists only of heated gas, it emits very little radiation, although it is the hottest part of the flame.

330. If the gas be mixed with air before being ignited, as in the flame of a Bunsen or gauze burner, the solid particles of carbon do not exist in it for a sensible time, and very little radiation is emitted. If the hand be held by the side of a Bunsen flame, and the supply of air be suddenly cut off so as to make the flame bright, the sensation of heat experienced by the hand will be at once very sensibly increased, owing to the increased radiation.

Even in a blow-pipe flame, where the air and gas are

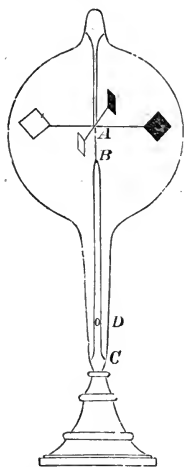
intimately mixed, the flame is not solid right up to the jet. It contains within it a space in which combustion has not yet commenced, and just beyond this space is the region where the greatest heat is generated, because there the actual union of the carbon and hydrogen with oxygen takes place. The reason of the existence of the uncombined though mixed gases in the middle of the flame is that there is a definite limit to the rate at which ignition can be propagated through a mass of gas. If the gas leave the jet with a greater velocity than this it will not ignite at once, but will only begin to burn where the velocity of the stream is not greater than the velocity of propagation of the temperature of ignition through the gas.

331. **The Radiometer.** In 1874 Mr. Crookes discovered that when radiation was received by a body in an almost perfect vacuum, it had a tendency to move in a direction opposite to that from which the radiation came; that is, to recede from the source of radiation. In his first experiments a small light body, such as a piece of elder pith, was attached to each end of a straw, through the middle of which half a needle, pointed at both ends, was passed, and the straw was supported in a horizontal tube upon the points of the half needle, which was a little shorter than the diameter of the tube. The balance was effected by heating the heavier end so as to dry it. When the tube was exhausted by a mercurial air-pump, it was at first found that a hot body appeared to attract the end of the straw near to which it was placed, but when the exhaustion reached a certain stage, which was different for different bodies balanced on the straw, the attraction ceased and repulsion set in. This repulsion was not due to currents of air, because it took place equally whether the hot body was placed below or *above* the straw. On substituting a piece of ice for the hot body, the ice appeared to attract the end of the straw near to which it was placed.

332. Mr. Crookes also found that if pieces of metal were balanced on a straw, the repulsion was greater when they were covered with lampblack than when polished,

and that generally the repulsion was greater the greater the absorbing power of the surface. From this it would follow that if a disc of metal were polished on one side and covered with lampblack on the other, and were exposed to radiation equally from all directions (as in diffused daylight), it would tend to move in the direction of the normal to its surface drawn outwards from its polished side; that is, the polished side would always appear to advance and the black side to retire. Upon this principle Mr. Crookes constructed the *Radiometer*, a modification of which is shown in Fig. 50.

FIG. 50.



It consists of a glass vessel into the bottom of which is sealed a tube, which is drawn off at the top, and carries a sharp steel point. On this rests a light glass cup, *A*, which carries four vertical discs of aluminium at the extremities of horizontal arms at right angles to each other. The discs are thus supported on a bearing similar to that of an ordinary compass needle. A very narrow glass tube attached to the top of the vessel prevents the cup from falling off the point if the instrument be inverted. The vessel is exhausted through the tube *BC*, in which a hole *D* has been formed for this purpose, and the end of the tube is then sealed. The discs are covered with lampblack on one side only, and are arranged in order, so that the black sides all point in the same direction round a horizontal circle. From what has been said it will be seen that, with this arrangement,

the arms carrying the discs will rotate when the latter are exposed to radiation, in the direction in which the polished faces look, in the same way as the arms of an anemometer carrying Robinson's cups rotate in the direction in which the *convex* surfaces of the cups look. The rate at which the arms revolve may be taken to indicate the intensity of the radiation to which the instrument is exposed, and hence its name.



333. It appears at first sight as if the action of the radiometer might be due to the impact of material particles, and thus seem to favour the corpuscular theory of radiation; but were the corpuscular theory true, the pressure on the polished surfaces at which reflection takes place, and from which the particles must therefore rebound, would be greater than that upon the blackened surfaces, which must retain them, and the arms would revolve in the direction opposite to that in which they actually move.

334. Perhaps the best explanation hitherto given of the repulsion produced by radiation is that due to Professor Osborne Reynolds, of which the following is a brief sketch.

Although a very good vacuum may have been produced within the receiver, there are always present some particles of air or other gas which, according to the dynamical theory of gases, are moving about in all directions, with velocities which increase with the temperature, and which exert pressure upon any surface exposed to their impacts. Now the surface of any body exposed to radiation becomes heated thereby, and the more so as its absorbing power is increased. Hence, in the radiometer the blackened sides of the vanes become heated more than the polished sides. When the particles of air strike the surface, they rebound with velocities which on the average are greater than those with which they strike it if the temperature of the surface be higher than that of the enclosure, and thus act like elastic balls, whose coefficient of elasticity is greater than unity. The greater the velocity with which they rebound, the greater is the pressure they exert on the surface, and this pressure will therefore be greater the higher the temperature of the surface. The pressure of the air on the side of a body exposed to a source of radiation will therefore be greater than that on the opposite side, and the body will consequently appear to be repelled by the source; while in the radiometer the pressure of the air on the blackened faces will be greater than that on the polished faces, because their temperature is higher, and the arms will therefore rotate in the direction in which they are observed to move.

335. If the radiometer be exposed for some time to a high temperature, so as to heat the vanes considerably above the temperature of the air in the room, and be then allowed to cool, the blackened sides of the vanes will cool most readily, being better radiators than the other sides, and the bright sides being thus left at the higher temperature, the vanes will rotate in the opposite direction to that in which they move when exposed to sunlight in the usual way. By concentrating sunlight by a lens on the vanes they rotate rapidly, the black sides retiring; then, on removing the instrument into the shade, the rotation ceases, and presently commences in the opposite direction.

336. By employing iron vanes, and floating the instrument in water, while the vanes were held stationary by magnets, Dr. Schuster found that the glass bulb rotated slowly in the opposite direction to that in which the vanes tended to rotate; thus showing that the stresses producing the motion were not between the vanes and the source of light, as would be the case if the light acted directly upon the vanes, but both *action and reaction* were confined to the instrument itself.

Clerk Maxwell, in 1879, gave a very complete mathematical investigation of the action of the radiometer, explaining why the vanes do not turn until the pressure of the air in the instrument is reduced below a certain limit.

## CHAPTER X.

### METEOROLOGY.

337. **Land and Sea Breezes.** We have now to consider the application of some of the experimental results we have obtained to the explanation of natural phenomena. We have seen that if air be heated without increasing the pressure to which it is exposed, it expands considerably. Hence hot air will rise in cold air on account of the action of gravity. Now, during a hot day the sun heats the surface of the land and of the sea, but owing to cooling by the evaporation of the latter, and communication of heat to the water below, on account of the continual movements of the sea, while the land is for the most part a very bad conductor of heat, the surface of the sea, and consequently the air in contact with it, becomes much less heated than that of the land. The air above the land being therefore more highly heated than that over the sea, will ascend, and air from the sea will flow in to occupy its place, forming a *sea breeze*.

During the night the land parts with its heat by radiation, and becomes much more cooled than the sea, on account of the low radiating power and high specific heat of the latter. The wind then blows towards the sea, since the air above the sea ascends, thus forming during the night a *land breeze*.

338. **The Trade Winds.** The equatorial regions of the earth become highly heated by the sun, which is vertical, or nearly so, in these regions at noon. The air there becomes consequently less dense than that in the adjoining temperate zones, and therefore ascends, to give place to the colder air, which, under the action of gravity, flows in from north and south, thus tending to produce a

north wind on the borders of the temperate and tropical zones in the northern hemisphere, and a south wind in the corresponding regions of the southern hemisphere. Now, the surface of the earth near the equator has a velocity from west to east of more than a thousand miles an hour, on account of the earth's rotation, while places in higher latitudes have a less velocity. As the air, then, flows towards the equatorial regions, it is continually reaching places which have a greater velocity from west to east than those from which it comes; and, since it cannot immediately acquire this velocity, it has always a motion from *east to west* relative to the place over which it passes. Combining this with its previous motion towards the equator, we get a north-east wind in the southern hemisphere. These are the *trade winds*.

339. **The Counter Trades.** Now, the air which ascended at the equator must come down somewhere to take the place of the air which flowed towards the equator from higher latitudes, and make room for the air which continues to rise at and near the equator. This current of air on reaching the earth's surface has, of course, a motion towards the nearer pole, but since it comes from places whose motion, on account of the rotation of the earth, is greater than those to which it is travelling, it will always have a velocity relative to these latter places in the direction in which the earth is turning,—that is, from *west to east*,—and combining this motion with its motion towards the pole, we get a south-west wind in the northern hemisphere, and a north-west wind in the southern hemisphere. These are the *counter trades*.

340. **The Gulf Stream.** On account of the great amount of heat absorbed by water in evaporating, the presence of a large quantity of water will have a great effect in moderating the excessive heat of the climate in summer; and its high specific heat enables it to give out so much heat while being cooled in winter, as to considerably alleviate the cold in that season. The great equatorial current which (probably partly driven by the trade winds) has a motion from east to west in the Atlantic Ocean, and rushing into the Gulf of Mexico

washes round its shores, and then crosses the Atlantic as the Gulf Stream, has much to do with moderating the winter temperature in the north-west of Europe. Places in central Asia and North America in the latitude of London experience excessive cold in the winter. The latitude of Moscow is not very different from that of Edinburgh.

341. **Clouds.** We have seen that if air be allowed to expand, by diminishing the pressure upon it, without allowing it to receive heat, it becomes cooled. Now, if the air be originally saturated with aqueous vapour, the diminution of the pressure which the vapour can exert on account of the decrease of temperature proceeds faster than the diminution of the pressure which it would have to exert on account of the expansion of the air if it all remained as vapour. Hence, as the air expands, some of the vapour must be condensed. In this way a cloud is formed in the receiver of an air-pump as the exhaustion proceeds.

Consider an ascending current of air nearly saturated with aqueous vapour. As it ascends it becomes cooled, and soon the vapour within it is no longer able, notwithstanding its expansion, all to exist as vapour, and a portion of it condenses, forming a cloud, which consists of drops of water, so small that the resistance offered to their motion by the air is sufficient to prevent their falling through it except with an exceedingly small velocity. The cloud seen to form in front of the spout of a tea-kettle containing boiling water, or above the safety-valve of a steam boiler, consists of small drops of water; and not of steam which is invisible. In the case of a kettle in which the water boils violently there is true steam, which is invisible for some distance in front of the spout.

342. Cloud may also be formed by a warm wind blowing against the side of a cold mountain. Here the wind is cooled by actual contact with the mountain, as well as by having to ascend its sides and so expanding under diminished pressure. As the air leaves the mountain, the cloud sometimes evaporates, so that there *appears* to be a stationary cloud around the mountain top notwithstanding

the presence of a strong wind. The cloud is, however, continually being blown away and renewed.

From this we should expect mountains to exert a great influence on the condensation taking place in their neighbourhood; and it is a matter of observation that the rainfall in mountainous districts is, other things being the same, much greater than on extensive plains, while the districts lying to leeward of lofty ranges are sometimes rainless.

343. Another cause of condensation is the mingling of currents of air at different temperatures. The pressure which aqueous vapour can exert at the mean of two temperatures is considerably less than the mean of the pressures it can exert at those separate temperatures; and consequently, when two quantities of air at different temperatures mix, if each be originally saturated with aqueous vapour, a portion of this must be condensed.

Thus, the greatest pressure of aqueous vapour at  $5^{\circ}\text{C}$ . is about  $\cdot 257$  in. of mercury, while at  $15^{\circ}\text{C}$ . it is about  $\cdot 5$  in. The mean of these is  $\cdot 3785$  in., but the greatest pressure of aqueous vapour at  $10^{\circ}\text{C}$ . is only  $\cdot 36$  in. Now, suppose two equal masses of air, saturated with aqueous vapour at the temperatures of  $5^{\circ}\text{C}$ . and  $15^{\circ}\text{C}$ . respectively, to mix. The temperature of the mass will then be reduced nearly to  $10^{\circ}\text{C}$ ., and consequently a portion of the vapour will be condensed till its pressure is only  $\cdot 36$  in., instead of  $\cdot 3785$  in., or about 5 per cent. of the whole amount of vapour in the two masses of air will be precipitated to form cloud. During this condensation the vapour which is condensed gives up to the air the latent heat of evaporation, thus preventing it from being cooled quite to the mean of the two temperatures, and also diminishing accordingly the amount of precipitation.

In all these cases *cloud is formed by the cooling of the air below the DEW-POINT.*

When a cloud is formed near the surface of the earth, it forms a *fog* or *mist*. The importance of solid or liquid particles in the air to act as nuclei in the formation of cloud or fog has already been referred to.

344. **Rain.** When some of the small drops of water in a cloud unite to form larger drops, they fall more rapidly than the others, and meeting with the small drops as they fall through the cloud, they attach themselves to them, thus increasing their own bulk, and in consequence falling more rapidly. The union of the small drops of water of a cloud or fog thus produces *rain*.

The very great amount of evaporation taking place in the equatorial regions, and the strong upward current of air there produced, account for the enormous rainfall in those parts.

345. **Hail and Snow.** If, as the small raindrops fall, they pass through colder regions of the atmosphere, they become frozen, and thus fall as *hail*. Different opinions are, however, held with regard to the mode of formation of hail.

If a cloud exist at a temperature below  $0^{\circ}\text{C}$ ., or be formed by the mingling of a quantity of warm air with air so cold that the temperature of the mixture is below  $0^{\circ}\text{C}$ ., the condensed vapour will consist of minute crystals of ice. It is these small ice crystals which form *halos* round the moon on cold nights, and sometimes, especially in the early morning, produce beautiful halos round the sun. The union of these small crystals produces *snow* flakes, which grow as they descend by the aggregation of other crystals to their exterior.

We have stated (Art. 168) that water may under special circumstances be cooled below  $0^{\circ}\text{C}$ . without freezing. Now and then this happens to raindrops which fall as water, but as soon as they meet with any disturbing body they congeal, forming a solid sheet of ice over the surface of an umbrella, or any other body on which they may fall, although its temperature may have been initially above  $0^{\circ}\text{C}$ .

346. **Dew.** The earth in any locality emits radiation at a rate depending upon the nature and temperature of its surface. During the day it receives radiation from the sun, but during the night this is not the case, and a fall of temperature is the result. If the night be cloudy,

the clouds radiate to the earth almost as much heat as they receive from it, so that its temperature is not much reduced; but if the night be clear, the surface of the earth radiates into space, and receiving nothing in return, a cold night is the result.

The air very close to the earth, then, frequently becomes cooled below the dew-point, especially if the day has been warm, and a white mist is seen to cover the land, particularly where grass, or the like, is growing. This mist deposited on the surfaces of the earth, of plants, etc., is *dew*.

347. Very much more dew is deposited on grass and similar vegetable structures than upon the bare earth or stones. The reason of this is that the blades of grass are exceedingly good radiators, but very bad conductors of heat; they therefore become intensely cold, and the air around them being cooled by contact, deposits very much of its moisture upon their surfaces. Stones and the like are not such good radiators as blades of grass, while their conductivity is much greater, so that they receive heat by conduction from the ground below, to make up in part for that which they lose by radiation.

On account of the great radiating power but small conductivity of straw, the water in shallow vessels, if exposed upon a heap of straw, may be frozen during the night in hot regions of India.

The theory of dew was first clearly worked out by William Charles Wells, a London physician, and the student interested in the subject is recommended to consult his essay, edited by Mr. L. P. Casella, and published by Messrs. Longman & Co. in 1866.

348. **Frost.** When the temperature of the surface of the ground, or of other surfaces, is reduced by radiation below  $0^{\circ}\text{C}$ ., this temperature being also below the dew-point of the surrounding air, the aqueous vapour becomes frozen as it is condensed, and thus *hoar-frost* is produced.

*Black-frost* is produced when the aqueous vapour, instead of *subliming* into hoar-frost, is first condensed into liquid water and then frozen.



It has been stated (Art. 303) that dry air is much more diathermanous than air containing aqueous vapour. Hence, in sandy deserts, where there is very little vapour in the air, not only is the earth intensely hot during the day, but at night, through the unchecked radiation, it becomes intensely cold.

## CHAPTER XI.

ON THE CONSERVATION OF ENERGY AND THE CONNECTION BETWEEN MOTION AND HEAT. BACON. RUMFORD. DAVY. SÉGUIN. MAYER. JOULE. SPECIFIC HEAT OF AIR AT CONSTANT VOLUME. HEAT ENGINES. DISSIPATION OF ENERGY.

349. **Work.** *Force is said to do work when it moves its point of application.*

The work done by a force which remains uniform in magnitude and direction is measured by the product of the number of units of force, and the number of units of length through which its point of application is moved *in the direction of the force*; that is, through which the force *works*.

If the weight of 1 lb. be the unit of force, and 1 foot the unit of length, the work done by a force equal to the weight of 1 lb. in working through 1 foot is the unit of work, and is called a *foot-pound*.

For example, if a weight of 10 lbs. be allowed to fall from a height of 6 feet to the ground, since the earth exerts an attractive force upon it *in the direction of motion* equal to the weight of 10 lbs., the work done upon the body by gravity is 60 foot-pounds.

350. Work is said to be done *against* a force, when the point of application is made to move (by some agent) in the direction *opposite* to that in which the force acts, and is measured by the product of the number expressing the force and that expressing the distance moved over by its point of application in the direction opposite to that in which the force acts.

Thus, if the 10 lb. weight above mentioned be raised

from the ground to its original position at a height of 6 feet, 60 foot-pounds of work are done *against* gravity.

The work done in lifting 1 cwt. of bricks to the top of a scaffold 40 feet in height will be the same, viz. 4480 foot-pounds, whether the bricks be raised in a basket over a pulley along a vertical straight line through 40 feet, or whether they be carried in a hod up an inclined ladder, or wheeled in a barrow up planks at a small inclination to the horizon, although in this last case the whole distance through which they are moved is very much greater than 40 feet. We see, then, that in order to find the work done against gravity, we have simply to multiply the weight of the body in pounds by the vertical height through which it is raised in feet, and the product is the required number of foot-pounds of work, and a similar process will give us the work done by gravity on a descending body, the weight of the body and the *difference of level* between its initial and final positions being all that concerns us, the form of the *path* described having no influence on the result.

351. Hence, in the case of a force whose direction is constant, if the point of application of the force do not move in the same straight line in which the force acts, we must determine the whole distance through which it has moved parallel to this straight line, and if this be given, the work done by, or against, the force is independent of the path along which the point of application moves.

352. If a weight of 10 lbs. be suspended at a height of 6 ft. from the ground by a string passing over a *smooth* pulley, it will be able, in descending, to lift a weight less than 10 lbs., but differing from it by a quantity as small as we please, and attached to the other end of the string, through the same height; and we may therefore say that in virtue of its *position* relative to the earth, it is capable of doing 60 foot-pounds of work upon another body, or to speak more correctly, the system composed of the 10 lb. weight and the earth, is capable of doing 60 foot-pounds of work in virtue of the relative positions of these bodies.

Similarly, if a spring be compressed it is capable of doing work (in virtue of its compression) when allowed to resume its natural form.

353. Again, if we succeed in catching a cricket-ball in one hand, we are conscious of a pressure exerted by the ball on the hand, and we also observe that the ball compels the hand to move backwards over a certain distance in the direction in which the ball is moving; that is, in the direction in which it presses the hand. Hence, the ball does an amount of work on the hand which, if the pressure exerted by it were uniform, would be measured by the product of this pressure and the distance through which the hand moves in the direction of the pressure. The ball is therefore capable of doing work *in virtue of its motion*. If the ball were to fall on the top of a spring, it would bend the spring by doing work upon it, and would thus put the spring in a position to do work on being allowed to regain its original form. (It may be shown that the amount of work the ball can do is proportional to the product of its mass and the square of its velocity.)

Of course the cricket-ball is unable to do work unless it meets with some object not moving with the same velocity as itself; so that, in strictness, we ought to say that the system consisting of the ball and the object which it strikes is capable of doing work in virtue of the relative motion of the parts. When we speak of the work which a moving terrestrial body can do, we generally mean the work it can do in being brought to rest relatively to the earth.

354. Hence we see that a system of bodies may be capable of doing work in virtue of its *configuration*; that is, the relative positions of its parts, or of *its motion*. The whole amount of work which a system is capable of doing is called its *energy*. That portion of its energy which is due to its motion is called its *kinetic energy*, while that portion of the energy which is due to its configuration called *potential energy*.

355. Potential energy and kinetic energy are mutu-

ally convertible. Thus, if a weight of 10 lbs. be held at a height of 6 ft. above the ground, it may be said to possess 60 foot-pounds of potential energy in virtue of its position relative to the earth. If it be allowed to fall *freely* from this height, at the instant it reaches the ground its potential energy is zero, but it has a considerable velocity, and the kinetic energy it possesses in virtue of this motion is exactly equivalent to 60 foot-pounds. If the velocity were now reversed, this kinetic energy would be at first the same as before, but the body would now continue to rise, losing velocity, and therefore kinetic energy, as it goes, until it would come to rest at a height of 6 ft. from the ground, where its kinetic energy would be zero, since it would be at rest, but its potential energy would be 60 foot-pounds. Of course if it remained unsupported it would immediately begin to descend again, and the same cycle of events might be repeated; so that we should have the potential energy originally possessed by the ball alternately converted into kinetic energy, and this reconverted into potential energy. This actually takes place in the case of the motion of a clock pendulum, or any *vibrating* body, the energy of which is *potential* when at the extremity, and *kinetic* when in the middle, of its swing. At any intermediate point the energy is partly potential and partly kinetic. The energy of a bent spring is potential energy, while that of a cannon-ball after its discharge is kinetic energy.

356. It is possible that if we knew more about the constitution of matter and the machinery by which one particle acts on another at a distance, we should have no need for this distinction between kinetic and potential energy. At first sight the energy of a quantity of compressed air, or of the high-pressure steam in a boiler, appears to be like that of a compressed steel spring, and we should class it as potential energy. But the dynamical theory of gases teaches us that this energy is due to the motion of the gaseous particles, the pressure of a gas being simply due to the impacts of the molecules upon any surface exposed to them. Hence we must regard the energy of compressed gases as kinetic energy, and perhaps if we

knew as much about the nature of solids and liquids as we think we know about gases, we might see a reason for regarding all kinds of energy as kinetic energy.

**357. Absolute Units.** It should be observed that the units of force and of work we have adopted, viz. the weight of a pound, and the foot-pound, both belong to a class of variable units called *gravitation units*, for a pound is properly a definite *quantity of matter*, and its *weight* depends upon the locality in which it is placed, as well as its height above the sea-level, since the intensity of gravity varies both with the latitude and the distance from the earth's centre. On this account these units are not well adapted for general use throughout the world, and other units, called absolute units, which do not depend on the attraction of the earth, have been employed to supersede them.

*The British absolute unit of force is that force which, acting on a pound of matter for a second, generates in it a velocity of one foot per second, and is called a poundal. The British absolute unit of work is the work done by this force in working through one foot, and is called a foot-poundal.*

This unit of force is rather less than the weight of half an ounce in the latitude of London.

The *power* of an agent is the rate at which it can work. The British absolute unit of power is that of an agent which can work at the rate of one *foot-poundal* per second.

An agent which can work at the rate of 33,000 *foot-pounds per minute*, or 550 *foot-pounds per second*, is said to be of one horse-power. It will be noticed that the horse-power is a gravitation unit, not an absolute unit, of power.

The term horse-power is often used in connection with the rate at which an engine or other agent is actually working, and without reference to the greatest rate at which it can work.

**358.** The units of length, mass, and time generally employed in scientific work are the centimetre, gramme, and second, and units based upon these are frequently distinguished by the letters C. G. S.

*The C. G. S. unit of force is that force which, acting on a gramme for a second, generates in it a velocity of a centimetre per second and is called a dyne.*

*The C. G. S. unit of work is the work done by a dyne in working through a centimetre, and is called an erg.*

The weight of a gramme at the sea level in Paris is equal to about 981 dynes.

Since the erg is an extremely small unit of work, being equal to little more than the work done in lifting a milligramme through a centimetre, it is convenient to employ a larger unit for practical purposes. The *joule* is such a unit, and is equal to 10,000,000 ergs.

*The C. G. S. unit of power is the power of an agent which can work at the rate of an erg per second.*

A rate of working equal to 10,000,000 ergs per second is called a *watt*.

**359. Conservation of Energy.** Newton, in commenting on the third law of motion, makes a statement which may be translated as follows:—

*“If the action of an agent be measured by the rate at which it works, and the reactions of the resistances, whether arising from friction, cohesion, or weight, by the rate at which work is done against them; and if amongst these reactions we include the rate at which kinetic energy is being generated in the system, action and reaction in all combinations of machines are equal and opposite.”*

This statement is equivalent to asserting that when an agent does work on a system, the whole of the work done is equivalent to the work done against gravitation, cohesion, friction (and other forces), together with the kinetic energy generated in the system. Now, we have seen that when work is done against gravity, an equivalent amount of potential energy is conferred on the body which is raised, and the same is true when work is done against any forces which, like gravity, are independent of time and of the motion of the system. Hence, if a system be acted on by forces of this nature only, Newton's statement is equivalent to the assertion that the work

done on the system by any agent is equivalent to the change produced by it in the sum of the potential and kinetic energy of the system. This is a case of the great principle of the Conservation of Energy. In particular, we see that *if no external agent affect the system, the sum of its potential and kinetic energy will remain constant.*

360. If work be done against friction, or force of a like nature, no potential energy is conferred on the moving system, for friction always acts so as to prevent motion, and can therefore have no tendency to bring a body back into its original position, as gravity tends to do when a weight is raised.

The work done against friction, therefore, appears at first sight to be lost, and for a long while this was believed to be the case. It has its equivalent, however, in the energy due to a motion which we have not yet considered.

361. If a brass button be rubbed vigorously on a board and then applied to the skin, we become conscious that the button is *hot*. The physical agent which produces this sensation is called *heat* (Art. 1). Now, the work done upon the button by the person rubbing it is not in this case converted into energy of position, as the button may be finally brought to rest at the point from which it started; nor is any portion of it finally converted into the energy of motion of the button as a whole, since it is at last brought to rest. It remains then for us to discover what has become of the work done on the button.

362. If a resined bow be rubbed against the string of a violin, both friction and adhesion tend to retard the motion, and work has to be done against these forces by the hand of the violinist. After the passage of the bow the instrument emits a musical note, and on examination it is found that the string is in a state of *vibration*, a state which it also communicates to the wood of the violin, whence most of the sound is emitted. In this case the work done by the hand of the violinist is partly transformed into the energy of the vibrating string and sound-board, and thence into the energy of sound-waves in air.



Again, if a number of elastic wires have each one end stuck into a board, like bristles in a brush, and the hand be made to pass over their tops, the work done by the hand will produce vibrations in the wires, which will be in part communicated to the board. The surface of any body which is not perfectly smooth, a condition fulfilled by no known object, consists of a series of small projections, and the passage of a rough surface over these causes them to vibrate, in the same manner as vibrations are set up by the passage of the hand over the wire-brush or of the bow over the violin-string, though of course on a very much smaller scale. Now, it is the passage of these projections on each surface over one another that is the cause of friction; and the work done against friction is consumed in setting up vibrations in these small portions of matter, these vibrations being continually communicated to the other material particles in contact with them (conduction of heat), so that vibrations are set up in every particle of which the button or other body is composed. (All natural surfaces possess projections of this nature; the art of polishing consists in making them as small as possible. A surface is highly polished when they are small compared with the length of a wave of light, or the 50,000th of an inch.) It is this state of vibration which, when communicated to our nerves, produces the *sensation of heat*, and the whole of the work done against friction is converted into the energy of vibration of the particles of which the bodies which rub against each other are composed. (The part played by local electric currents as an intermediate form assumed by the energy in this production of heat has not been determined.)

363. We can now place the conservation of energy upon a broader basis than before, comprehending within it those cases in which work is done against friction and similar non-conservative forces, and may enunciate it thus:

*When an agent does work on any material system, the whole of the work done is equivalent to the change of the sum of the potential and kinetic energy of the system, in-*

*cluding in the latter the energy of the vibrations set up in the molecules of which the system is composed. If the system do work against an external resistance, its loss of energy is equivalent to the work so done.*

We thus see that the work done against friction, and supposed by Newton to be lost, has its equivalent in molecular vibrations, constituting heat, generated in the system.

Maxwell stated the principle of the Conservation of Energy very elegantly as follows :

*"The energy of a system is a quantity which can neither be increased nor diminished by any actions between the parts of the system, though it may be transformed into any of the forms of which energy is susceptible."*

364. In the case of the vibrating violin-string, the vibrations are communicated by the sound-board of the instrument, and, in small part, by the string itself, to the surrounding air, through which they are propagated to distant objects, and their energy is lost by the violin. In a similar way the vibrations which, as we have said, constitute heat in bodies, are slowly imparted to a medium which fills space, viz., the ether, and are propagated by it to distant bodies in the form of radiant energy, which may or may not be capable of producing the sensation of light, and energy is then lost by the body in which it originally existed as heat. It is the communication of vibrations *from particle to particle of a body* which constitutes the *conduction* of heat.

365. It was formerly believed that heat consisted of a kind of imponderable fluid which pervaded all bodies, and that when heat was produced by the blow of a hammer, or the sudden compression of air, it was *squeezed out* of the body operated upon by the pressure to which it was exposed.

Long before the Dynamical Theory of heat was established, Lord Bacon expressed his conviction that heat consists of a kind of motion or "brisk agitation" of the particles of matter. In the *Novum Organum*, after giving a long list of sources of heat, in which he mentions the

friction of trees rubbing against one another in a high wind, and other mechanical sources, he says: "From these examples, taken collectively as well as singly, the nature whose limit is heat, appears to be motion." Again, "It must not be thought that heat generates motion or motion heat, though in some respects this is true, *but the very essence of heat, or the substantial self of heat, is motion and nothing else.*" But Bacon was too apt to theorise without sufficient experimental evidence, and the following quotation almost spoiled what he had previously said: "Comparing the effects of fire with those of time; time dries, consumes, undermines, and reduces to ashes as well as fire, and perhaps to a much finer degree; but as its motion is very slow, and attacks very minute particles, no heat is perceived." He also confused the acrid and irritant properties of acids, essential oils, and the like, with heat, and though he cites many observations which afford very strong evidence in favour of the Dynamical Theory of heat, yet we must give to Rumford and Davy the credit of having established it on a scientific basis.

**366. Rumford's Experiments.** Benjamin Thompson, Count Rumford, was engaged by the Bavarian Government at the military arsenal in Munich. While superintending the boring of cannon, he was struck by the amount of heat produced by the action of the boring-bar on the gun-metal castings, and this induced him to determine, if possible, whether or not there existed an igneous fluid or caloric to which the sensation of heat was due. To account for the heat developed by the boring-bar, the first source which presented itself as that from which the heat might have been derived, was the metal abraded by the borer. If this were the case, he argued, the capacity for heat of the chips ought to differ from that of the solid metal by an amount sufficient to account for the whole of the heat generated. To ascertain whether this were the case, he took equal quantities, viz. 1016½ grains, of the borings and of thin slips cut from the same block of metal with a fine saw. The slips of metal were first heated to 210° F. and then plunged into water

at a temperature of  $59\frac{1}{2}^{\circ}$  F., contained in a cylindrical tin vessel. After one minute the temperature of the water was  $63^{\circ}$  F. The tin vessel and water within it had together the same capacity for heat as 4590 grains of water. From these data it will be seen that the specific heat of the metal was about .11. The same experiment was then tried with the metallic chips, and the same result obtained; moreover, each experiment was repeated three times with nearly the same results. From this Rumford concluded, "that the heat produced could not possibly have been furnished at the expense of the latent heat of the metallic chips."

367. Rumford next turned up a hollow cylinder which was cast in one piece with a brass six-pounder, and separated from it by a narrow neck of metal. The weight of this cylinder, exclusive of the neck and the rest of the casting, was 113.13 lbs. A blunt steel borer exerted a pressure against the bottom of the cylinder equal to the weight of about 10,000 lbs. After the cylinder had made 24 revolutions per minute for 30 minutes, its mean temperature was found to be  $130^{\circ}$  F., while its temperature before the operation was  $60^{\circ}$  F. The borings detached from the metal during the process amounted to only 837 grains.

368. It then occurred to Rumford that this heat might possibly be due to the action of the air, and in order to test this he surrounded the small cylinder by a wooden box, through one side of which the square boring-bar passed, while the narrow neck which connected the cylinder with the cannon fitted a circular hole in the opposite side of the box, in which it could turn freely. About 18.77 lbs. of water were placed in the box, so as to completely cover the cylinder, and the brass casting was made to turn at the rate of 32 revolutions per minute by the action of two horses, though Rumford remarks that one horse would have been sufficient for the work. The temperature of the water at the commencement of the experiment was  $60^{\circ}$  F.; in  $2\frac{1}{2}$  hours it boiled. Taking into account the heat required to raise the temperature of the gun-metal cylinder and of that part of the boring-

bar which was within the water, Rumford estimated that the heat produced was sufficient to raise 26·58 lbs. of water from the freezing-point to the boiling-point, and this calculation neglected the heat absorbed by the box and that dissipated by conduction and radiation. The quantity of metal borings produced during the experiment amounted to 4145 grains. From this result Rumford concluded that the work of one horse was capable of generating heat more rapidly than nine ordinary wax candles burning in the usual manner.

369. Finally Rumford reviewed the sources from which the heat might, *a priori*, have been thought to have been derived. That it could not have come from the borings was shown by the first experiment. It could not have been due to the action of the air, for it was produced just as readily when the whole apparatus was filled and surrounded with water. It could not have come from the water, for this received heat, and no chemical or other change took place in the water itself to account for the heat generated. That the heat should have come from the boring-bar or neck of metal connecting the cylinder with the gun appeared more improbable than the previous hypotheses, for throughout the experiment heat was escaping from the apparatus by each of these channels. Moreover, "the source of the heat generated by friction in these experiments appeared evidently to be inexhaustible."

"It is hardly necessary to add, that anything which any insulated body, or system of bodies, can continue to furnish *without limitation*, cannot possibly be a *material substance*; and it appears to me to be extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated in the manner the heat was excited and communicated in these experiments, except it be *MOTION*." (Rumford.)

370. **Davy's Experiment.** About the same time as Rumford's experiments were published (1798), Sir Humphry Davy showed that if two pieces of ice were made to rub against one another, even in a vacuum, though everything surrounding them were at a tempera-

ture below the freezing point, the pieces of ice could be melted by the friction. In this case the heat could not have come from the ice, for heat is absorbed by it in fusing. Davy did not, however, see at the time that this experiment afforded conclusive evidence against the theory of the material nature of heat.

371. The experiments of Rumford and Davy showed that an unlimited amount of heat could be obtained by purely mechanical means, and completely disproved the material or caloric theory of heat. The researches of Joule showed that when heat is produced by mechanical means, no matter how complicated the mechanism may be, or what transformations of energy take place, the quantity of heat produced is *always proportional to the quantity of ENERGY expended in producing it*; or the production of one unit of heat always requires the performance of the same number of units of work. Since for each unit of heat produced, the same quantity of energy, in some form or other, disappears, no matter how the process may be conducted, it follows that energy is the mechanical quantity which corresponds to heat, the other forms of energy being actually converted into heat. Thus heat is only one of the many forms of which energy is susceptible.

372. **Mechanical Equivalent of Heat.** DEF. *The quantity of energy which, if entirely converted into heat, is capable of raising the temperature of the unit mass of water from  $0^{\circ}$  C. to  $1^{\circ}$  C., is called the mechanical equivalent of heat.*

Séguin, a nephew of Montgolfier, was one of the first to attempt to measure the mechanical equivalent of heat. He argued that, if heat be energy, when a steam-engine does work some of the heat of the steam must be consumed, and, consequently, less heat will leave the engine than if the same quantity of steam were blown through without doing any work. Hirn, in 1857, succeeded in detecting this difference experimentally, and also in measuring it. From his result, knowing the amount of work done by the engine, he calculated approximately the mechanical equivalent of heat.

373. Séguin, in 1837, endeavoured to determine the mechanical equivalent of heat from the loss of heat suffered by steam in expanding, but he assumed that the whole of this heat was consumed in doing work against the pressure to which the steam was exposed, taking no account of the change in the condition of the steam itself.

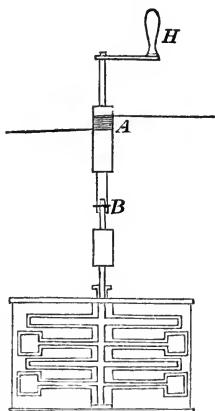
374. In 1842, Mayer, a physician at Heilbronn, published an attempt to determine the mechanical equivalent of heat from the heat generated in the compression of air; but he made the same assumption as Séguin. Joule subsequently showed that when air is compressed nearly the whole of the work done is converted into heat; but this Mayer had no right to assume without experimental evidence. On account of errors in his values of the specific heats of air at constant pressure and at constant volume, Mayer's result was considerably in error.

The omission made by Mayer and Séguin in their reasoning on this subject may be illustrated by taking a somewhat extreme example. Suppose energy expended in compressing a helical steel spring; a certain amount of heat will be generated on account of the imperfect elasticity of the steel; but this will absorb but a very small fraction of the energy expended in compressing the spring. The rest of the energy will be expended in changing the relative positions of the particles of steel in opposition to the forces which act between them. When the spring returns to its natural form this energy is again liberated, and may be employed in doing an amount of work nearly equal to that done in compressing the spring. If in the case of air or steam there were forces acting between the particles tending to keep them in some definite position relative to one another, and to oppose any change of position, it would follow that on compressing the material, more or less of the work done would be expended in doing work against the molecular forces, and would be stored as potential energy in the compressed substance to be liberated in doing external work when the pressure is relieved. The amount of heat (kinetic energy) produced by the compression would consequently

be less than the equivalent of the work done in compression by the amount of potential energy so accumulated. When Joule showed that no potential energy was liberated when air expanded into vacuum (Art. 387), he also proved that no forces were exerted between the particles of air at their ordinary distances apart.

375. To Dr. Joule, of Manchester, belongs the credit of having first determined the mechanical equivalent of heat with great accuracy by a course of investigation which was logically complete. His best known experiments were carried out in 1849, and consisted in determining the amount of heat generated by the friction of a metal paddle in vessels containing water and mercury, and also by the friction of bevel wheels of cast iron against each other, the amount of work expended in driving the apparatus being measured. The arrangement employed to produce friction in water consisted of eight

FIG. 51.



paddles made to revolve in a cylindrical box of water between stationary vanes, as in an ordinary churn. The forms of the paddles and vanes are shown in Fig. 51. The paddles were made to revolve by a weight which in its descent turned a roller, around which part of a string was wound, another part of the same string being wound round the cylinder *A*, which could be coupled by a pin *B* to the paddle. When the weight had run down it was wound up by turning the handle *H* after removing the pin *B*, when, the coupling being removed, the paddle was unaffected by the motion of the handle. The mass of

the driving weight being known, as well as the whole distance through which it fell, the whole amount of work done upon the apparatus was determined in foot-pounds, and allowance made for all resistances outside the box containing the water, and for the small amount of kinetic energy possessed by the weights when they struck the



ground. The temperature of the water in the vessel was observed during the experiment, and thermometers which could be read to  $\frac{1}{200}^{\circ}\text{F.}$  were employed; the amount of water, as well as the capacity for heat of the vessel and paddle, were carefully measured, and allowance made for conduction. These experimental data are all that are required for the determination of the mechanical equivalent of heat, and the results showed that *the amount of work which must be entirely converted into heat, in order to raise the temperature of one pound of water  $1^{\circ}\text{C.}$  from  $0^{\circ}\text{C.}$ , is  $1390.846$  foot-pounds.*

376. The apparatus employed by Joule for stirring mercury was similar to that described in the last article, but somewhat smaller and made of iron. In another set of experiments he employed cast-iron wheels, bevelled so as to form portions of a hollow and of a solid cone, and these were made to grind against one another, the solid cone being pressed into the other by means of a lever, and the rotation kept up by a falling weight. Two sets of weights were employed in different experiments, the larger pair being about 58 lbs. and the smaller  $19\frac{1}{2}$  lbs.

377. The extreme results of all these experiments varied by little more than one-half per cent., although the experiments differed so widely in their details, and from them Joule inferred that "the amount of heat produced by friction is proportional to the work done and independent of the rubbing surfaces," and that the mechanical equivalent of heat is about 1390 foot-pounds.

Hence, according to Joule,

*If the amount of heat required to raise a pound of water from  $0^{\circ}\text{C.}$  to  $1^{\circ}\text{C.}$  be taken as the unit of heat, the mechanical equivalent of heat is 1390 foot-pounds.*

This is usually denoted by J.

If a quantity of water fall freely through 1390 feet, the heat produced in bringing it to rest would raise the temperature of the water through  $1^{\circ}\text{C.}$

378. The experiments just cited afford the most satisfactory determinations of the mechanical equivalent of

heat, but were not the first of Joule's experiments on the subject. In 1840 Joule showed that if a magneto-electric machine were employed to produce an electric current, and the energy of the current entirely converted into heat in the conductors through which it passed, the amount of heat so produced was the same as if the energy employed in driving the machine had been directly converted into heat by friction or other mechanical means, without the intervention of the current. He also showed that the whole heat generated in a voltaic circuit was proportional to the work done by the electro-motive force of the battery.

379. Knowing the mechanical equivalent of heat, we can determine the amount of heat generated in any mechanical action if we know the number of foot-pounds of energy transformed.

For example, the amount of heat which would be produced by the fall of 100 tons of stone through a vertical height of 1000 feet down the side of a mountain, supposing the mass to come to rest at the end of this distance, would raise the temperature of  $\frac{2240 \times 100 \times 1000}{1390}$  pounds, or, 161,151 pounds, of ice-cold water  $1^{\circ}\text{C.}$ , or would convert about 253 lbs. of water, initially at  $0^{\circ}\text{C.}$ , into steam at  $100^{\circ}\text{C.}$

(In this case, as explained above, the potential energy initially possessed by the earth and stone, on account of their relative positions, is converted into kinetic energy in the fall, and this into heat or thermic energy, by striking the ground.)

380. **Specific Heats of Air.** We have seen that work may be readily converted entirely into heat. Heat must therefore be of the same nature as work—that is, a form of energy; and by means of its mechanical equivalent, quantities of heat and of other forms of energy become comparable.

As work can be converted into heat, so, by proper appliances, heat may be converted into work. An appliance for this purpose is called a *heat-engine*. Steam-engines,

hot-air engines, gas-engines and the like, are forms of heat engines. The conversion of heat into work may be shown by taking a quantity of air compressed in a tall cylinder by a number of weights placed upon a piston which fits it. By removing the weights one by one the air will expand, and in doing so will lift the weights left on the piston, and therefore do work. If the weights be removed in sufficiently quick succession, the air will be found to be cooled by its expansion. Now, Joule showed that when air expanded into vacuum, and consequently did no external work during its expansion, its temperature was on the whole unaffected. Hence, the fall of temperature of the air lifting the weight must be due to a conversion of part of its heat into work.\* Again, it is found that more heat is required to raise the temperature of a mass of air through a given number of degrees when it is allowed to expand against a constant pressure than when its volume is kept constant; in other words, the specific heat of air at constant pressure is greater than its specific heat at constant volume. But its temperature is unaffected by change of volume when it does no external work, though no heat is allowed to enter or leave it. Hence, the difference between the two specific heats must be due to the heat converted into external work when the gas is allowed to expand.

381. The experiment by which Joule showed that the whole of the heat absorbed by a gas expanding at constant pressure was consumed in doing external work, was undertaken in order to justify one of his modes of determining the mechanical equivalent of heat. Air at a pressure of 22 atmospheres was allowed to escape from a vessel through a coil of very fine lead tube, and finally to escape into the air. The vessel containing the compressed air was placed in a calorimeter, as was also the tube, and the heat produced by the escape of the air measured.

\* The molecular theory of gases completely explains this result, because when the piston is rising it is moving in the direction in which the particles of gas strike it; they therefore rebound with diminished velocity, and although they behave like *perfectly* elastic balls, they lose energy, that is, they lose heat, by the impact.

The energy required to compress the air in the vessel could be easily calculated, and to complete the determination it was only necessary to know that no heat is consumed by air in expanding at constant temperature when it does no work. To test this, Joule connected together two equal vessels, one containing air at a pressure of 22 atmospheres, and the other exhausted. The vessels were placed in a calorimeter, and on opening the communication between the two, so as to allow the air to expand, no change took place in the temperature of the water. When the two vessels were placed in separate calorimeters, the water in that containing the compressed air vessel became cooled, while that in the other became heated to the same extent.

382. From what we have said, it will appear that the mechanical equivalent of heat enables us, if we know the specific heat of air at constant pressure, to determine its specific heat at constant volume. The method will be made clear by the following example :—

Suppose a cubic foot of air under a pressure of 15 lbs. weight per square inch and at  $10^{\circ}\text{C}$ . to weigh  $1\frac{1}{4}$  oz., and suppose its specific heat at constant pressure to be  $\cdot 237$ . Let the air be placed in a cylinder closed by a piston whose area is one square foot, the weight of the piston being so counterpoised that the pressure on the air in the cylinder is equal to the weight of 15 lbs. on each square inch. Then the whole pressure of the air on the piston will be equal to the weight of  $144 \times 15$  lbs., that is, of 2160 lbs. Now let the air be heated to  $100^{\circ}\text{C}$ . It will then expand (Art. 132) till its volume is  $\frac{373}{283}$  cubic

feet, and will therefore raise the piston through  $\frac{90}{283}$  feet.

Hence, the work done by the air in expanding will be  $\frac{2160 \times 90}{283}$  foot-pounds, or about 687 foot-pounds. The

mechanical equivalent of heat being 1390 foot-pounds, the amount of heat consumed in doing this work would raise the temperature of  $\frac{2160 \times 90}{283 \times 1390}$  pounds of water  $1^{\circ}\text{C}$ .

Now, the mass of air heated is  $1\frac{1}{4}$  oz. or  $\frac{5}{64}$  of a pound, and its temperature is raised through  $90^{\circ}$  C., its specific heat being  $\cdot 237$ . The amount of heat employed would therefore raise through  $1^{\circ}$  C. the temperature of  $\frac{5 \times 90 \times \cdot 237}{64}$  lbs. of water, or of 1.666 lbs. of water. But

of this number of units of heat,  $\frac{2160 \times 90}{283 \times 1390}$ , or  $\cdot 4942$  units are consumed in doing external work. Therefore the amount employed in actually raising the temperature of  $\frac{5}{64}$  lb. of air through  $90^{\circ}$  C. is  $1.6666 - \cdot 4942$ , or  $1.1724$  units. But since air is not cooled by expansion if it do no external work, this amount is the same as would be required to raise the temperature of the same air to the same extent if its volume were kept constant. Hence, to raise the temperature of  $-\frac{5}{64}$  lb. of air through  $90^{\circ}$  C. at constant volume, we require  $1.1724$  units of heat, and the specific heat of air at constant volume is therefore

$$\frac{1.1724 \times 64}{90 \times 5} = \cdot 1667...$$

From this we see that the ratio of the specific heat of air at constant pressure to its specific heat at constant volume is  $\frac{\cdot 237...}{\cdot 166...} = 1.42...$  According to the best experiments, the value of this ratio for air and other permanent gases is between 1.41 and 1.42.

**383. Heat Engines.** We have said that heat engines are capable of converting heat into work, and this is usually effected by taking advantage of the expansion of some substance when heated. This substance is called the *working substance*. If the engine, after a complete stroke, leaves the working substance in the same state in which it finds it, so that it is capable of employing the same substance an indefinite number of times in succession, the substance is said to go through a complete *cycle* of changes; as, for instance, when the

waste steam from an engine is condensed and returned to the boiler.

384. An engine can convert heat into work only by allowing heat to pass into the working substance from a *source* at a high temperature, and allowing *part* of this to escape from the substance to a *condenser* at a lower temperature. Thus, the engine cannot convert the whole of the heat it receives from the source into work.

*The ratio of the amount of heat which an engine converts into work to the amount received from the source, is called the EFFICIENCY of the engine.*

In measuring the efficiency of an engine, the working substance is supposed to be left by the engine in the *same condition as to temperature and volume* as that in which it was found by it. Otherwise the engine might utilize some of the energy *contained in the substance itself*, and so leave it deprived of part of its energy, as when air does work in expanding under a continually diminishing pressure without receiving any heat from without.

385. If two bodies at different temperatures be placed in contact, heat will pass from the hot to the cold body without doing any work on the way. We might thus allow the whole of the heat from the source to pass into the condenser without doing any work. In order, then, to make the most of the heat supplied by the source, we must so arrange matters that when heat passes from the source into the working substance, or from the working substance to the condenser, the difference of temperature in each case is the least possible. The *efficiency* of the engine will then be a maximum.

386. **Absolute Scale of Temperature.** It can be shown that if this difference of temperature could be made indefinitely small, and no heat be allowed to leave the working substance by radiation or conduction, except that which enters the condenser, the efficiency of the engine would be very nearly proportional to the difference between the temperatures of the source and condenser divided by the temperature of the source, reckoned from the absolute zero of the air thermometer.

A scale of temperature for which this proportionality is exact instead of approximate is known as 'Thomson's absolute scale of temperature, having been first proposed by Sir William Thomson. The difference between it and the scale of the air thermometer, described in Art. 151, is almost insensible. This scale is independent of the properties of any particular substance, and is therefore entitled to be called the *absolute* scale of temperature. An engine which fulfils the above condition is said to be reversible, because by an indefinitely small change in the temperature of the working substance, the flow of heat and every action of the engine can be reversed, provided that an amount of energy be expended by an external agent which is exactly equivalent to the work done by the engine when working forwards for the same number of strokes.

**387. Reversible Engine.** *Suppose an engine, which satisfies the above conditions, to be supplied by heat from a source at  $250^{\circ}$  C., and to give out heat to a condenser at  $100^{\circ}$  C. Suppose also that the engine works at the rate of 10 horse-power; it is required to find the amount of heat taken from the source and the amount given out to the condenser per hour.*

The measure of the efficiency of the engine is  $\frac{250 - 100}{250 + 273}$ , or  $\frac{150}{523}$ . Now, the work done per hour is  $33000 \times 10 \times 60$  foot-pounds, or 19,800,000 foot-pounds, and this implies the conversion into work of  $\frac{19800000}{1390}$ , or 14244.6..., units of heat. But only  $\frac{150}{523}$  of the heat received is converted into work, while the remaining  $\frac{373}{523}$  of the heat is given out to the condenser. Hence, the number of units of heat received per hour from the source is  $14244.6 \times \frac{523}{150}$ , or 49666.1..., and the amount given out to the condenser is this quantity less that converted into work, viz.—35421.5... units.

388. **Dissipation of Energy.** From what we have said it will appear that our ability to convert heat into work depends on our power of commanding differences of temperature, and the *availability* of any quantity of heat for useful purposes depends upon the difference between the temperature of the body containing it and the lowest temperature we can command. Now, if we have a hot and cold body and a heat engine, part of the heat of the hot body can be converted into, or is available for, work; but if the bodies be placed in contact, they will eventually attain the same temperature, and though no heat has been lost by the two together, yet that which they contain is no longer available for conversion into work without the introduction of a colder body. If we could obtain no differences of temperature, heat engines would have no existence. Thus, although the whole amount of energy possessed by a system of bodies unacted upon by anything from without remains unaltered, yet the *availability* for doing external work of this energy is continually diminishing. The principle involved in this statement is called the principle of the *Dissipation of Energy*.

389. The following example is interesting since it fairly represents the working of some of the best condensing engines of a few years ago.

*An engine consumes  $1\frac{3}{4}$  lb. of coal per indicated horse-power per hour. The heat developed by the combustion of 1 lb. of coal is capable of converting 15 lbs. of water at  $100^{\circ}$  C. into steam at the same temperature. Find what fraction of the whole heat generated is employed in doing work in the engine.*

To evaporate 15 lbs. of water requires  $15 \times 537$ , or 8055, units of heat. Hence,  $1\frac{3}{4}$  lb. of coal develops by combustion  $14,096\frac{1}{4}$  units of heat, and this amount is equivalent to  $14,096\frac{1}{4} \times 1390$ , or  $19,593,787\frac{1}{2}$ , foot-pounds of energy.

But one horse-power sustained for one hour means  $33,000 \times 60$ , or 1,980,000, foot-pounds of work. The ratio of the work done to the mechanical equivalent of the heat



developed—that is, the efficiency of the engine—is, therefore,  $\frac{1980000}{19593787\frac{1}{2}}$ , or about 10·1 per cent.

The engine however is not responsible for the whole of the heat wasted. At least 30 per cent. of the heat is lost through the inefficiency of the boiler.

390. If the temperature in the boiler be 150° C., and that in the condenser 30° C., we may compare the efficiency of the engine with that of a *perfect* engine working between the same limits of temperature, thus:—

The efficiency of the perfect engine will be denoted by  $\frac{120}{423}$  or  $\frac{40}{141}$ . The efficiency of the engine in question is about ·101. The ratio is therefore  $\cdot 101 \times \frac{141}{40}$ , or about ·356.

This shows that considerable improvement may yet be effected in the efficiency of steam-engines before we reach the theoretical limit corresponding to the temperature of our boilers.

Triple expansion engines, working with steam at 160 lbs. pressure, will give an indicated horse-power for 1·2 lbs. of coal per hour.

391. From what has been said, it appears that the efficiency of an engine may be expected to improve if the difference of temperature between the boiler and the condenser can be increased, and that for the same difference of temperature the efficiency will be greater the lower the temperature of the boiler. Hence, condensing engines are more efficient than engines in which the steam blows off into the air at a temperature above 100° C.

With a condensing engine it is not easy to keep the temperature of the condenser below about 30° C. Hence, to keep the difference as great as possible, we must raise the temperature of the boiler. The diminution of the tensile strength of the boiler plates at high temperatures, and the injury inflicted on the cylinder, lubricants, and packing, by steam at very high temperature, however, assign a pretty definite limit to the boiler

temperature, and it is very seldom indeed that a temperature much exceeding  $200^{\circ}\text{C}$ . is permitted.

An inferior limit to the temperature of the condenser is, of course, afforded by the temperature and quantity of the water available for condensing, but a higher limit is frequently assigned by the fact that the condensed steam becomes the feed water, and is forced back into the boiler. If the feed water enters the boiler at too low a temperature, the local cooling produced has a very injurious effect on the plates. In consequence of this, as well as to avoid the necessity of employing very large quantities of condensing water, it is not uncommon to keep the temperature of the hot well, and therefore of the interior of the condenser, at about  $50^{\circ}\text{C}$ .

**392. The Separate Condenser.** In Newcomen's pumping engine, steam was employed beneath the piston simply to raise the piston against the pressure of the atmosphere, and at the same time the weight of the pump rods suspended from the other end of the beam forced up the water from the mine or well as the rods descended. Cold water was then injected into the cylinder condensing the steam, and the atmospheric pressure forced down the piston, raising the pump rods in readiness for another stroke. But the cold water in the cylinder cooled the metal, and a large quantity of steam had to be blown into the cylinder to raise its temperature again to  $100^{\circ}\text{C}$ ., before the piston could be lifted for the next stroke. This entailed a great waste of heat. James Watt's greatest invention consisted in the application of the separate condenser, which is a chamber kept cool either by the application of cold water to its exterior (as in surface condensers) or by the injection of cold water into the chamber, or by both these methods. When the exhaust port is opened, the steam in the cylinder has free access to the condenser, and in accordance with the principle explained in the chapter on Evaporation, the pressure of the steam in the cylinder must fall to that due to the temperature of the coldest part of the enclosure to which the steam has access. The separate condenser is therefore as efficient in condensing the steam of the cylinder as

though the condensing water were injected into the cylinder itself, while the high temperature of the cylinder is maintained.

393. It is usual to cut off the supply of steam from the boiler long before the stroke of the engine has been completed, and then to allow the steam imprisoned in the cylinder to expand, and thus drive the piston through the rest of its stroke. In a high-pressure engine which exhausts only into the air, it would be useless to allow the expansion to continue until the pressure is less than atmospheric pressure, since it must be raised to atmospheric pressure when the exhaust port is opened; but in condensing engines the steam may be allowed to expand until the pressure is very little more than that due to the temperature of the condenser, and in some classes of engines (pumping engines, for example) it is not unusual to allow the steam to expand from a total pressure of, say, 60 lbs. on the square inch to a pressure of only 3 or 4 lbs. The temperature of the cylinder being kept considerably above  $100^{\circ}$  C., while its interior is in communication with the condenser (at, say,  $50^{\circ}$  C.) for half the time during which the engine is working, it follows that any water which finds its way into the cylinder will immediately distil over into the condenser, and the interior of the cylinder will remain practically dry. Hence, when the steam in the cylinder is allowed to expand, no evaporation takes place, so as to provide more steam to keep up the pressure, and the pressure falls very nearly in accordance with Boyle's law. To maintain the temperature of the cylinder, it is necessary that it should be very efficiently jacketed with steam at full boiler pressure.

394. In some steam turbines which have recently been constructed by Messrs. C. A. Parsons & Co., the pressure in the condenser is kept down so as to be little more than one inch of mercury, or less than half a pound on the square inch. In these turbines the steam actually expands in the working drum to 150 times its original volume or more, and is so cooled by the expansion that its temperature is scarcely above blood heat when it leaves the turbine. In the cast-iron case surrounding the low-

pressure blades there are inspection holes, which are covered with very thin brass plates to one side of which the steam has free access. The back of the hand can be laid on the other side with perfect comfort while the engine is running, and the warmth experienced is due to conduction from other parts of the case rather than from the steam itself; for the heavier the load on the engine, and the more steam there is consequently flowing through the drum, the cooler are the plates, showing that the steam carries off the heat which is brought to the brass plates by conduction from the high-pressure portion of the engine.

395. **Specific Heat of Saturated Steam.** If a quantity of saturated vapour be compressed, and no heat be allowed to escape from it, its temperature will be raised by the heat generated by the compression, and at the same time its pressure will be increased. It may happen that the increase of temperature more than compensates for the diminished volume and increased pressure, in which case the vapour will remain dry, and will, in fact, cease to be saturated (becoming superheated). On the other hand, it may happen that the increase of temperature does not compensate for the increased pressure, and in this case partial condensation will take place. The latter alternative is the case with alcohol vapour; but if steam be suddenly compressed so that no heat escapes, the consequent rise of temperature more than compensates for the increase of pressure, and the steam becomes superheated. In order therefore to keep the steam saturated, as its pressure is increased and volume diminished, it is necessary to abstract heat from the steam. Hence, if a quantity of saturated steam has its temperature raised and, at the same time, its pressure increased and volume diminished, so that it remains saturated, heat must be abstracted from it. In other words, that the steam may remain saturated, increase of temperature must be accompanied by loss of heat. Similarly, decrease of temperature, involving expansion and diminution of pressure, will necessitate the communication of heat to the steam if partial condensation is to be prevented.

These results are sometimes expressed by the somewhat curious statement that *the specific heat of saturated steam is negative*. This phrase, of course, simply implies that during compression the work done on saturated steam raises the temperature more than is consistent with saturation, and to preserve the steam saturated heat must be abstracted ; while the reverse is, of course, the case during expansion.

396. **Steam Jackets.** If saturated steam be allowed to expand, doing the full amount of work of which it is capable during the expansion, it loses so much heat that, notwithstanding its increased volume, partial condensation takes place. When this happens in the cylinder of an engine working expansively, some of the steam pressure must be lost unless means are taken to prevent condensation. One method consists in surrounding the cylinder with a steam jacket, which keeps up the temperature of the contents of the cylinder by heat conducted through the sides of the cylinder. The principal difficulty consists in causing the steam to take up the heat with sufficient rapidity.

If a quantity of air be allowed to expand and do work, and its temperature be kept constant meanwhile, an amount of heat will have to be given to it equivalent to the work done by it in expanding, and very nearly the same will be the case with steam. Hence, in a jacketed cylinder, if the temperature of the steam be kept constant throughout the stroke, the heat received from the steam jacket must be the equivalent of the whole work done during the expansion ; so that this amount of work is not obtained at the expense of the heat of the working steam, but at that of the steam employed in the jacket, and it is plain that, though loss of heat from the outside of the cylinder may be nearly prevented, a considerable amount of steam will have to be supplied to the jacket to furnish the heat required by the expanding steam within. Where the temperature of the boiler is not too high, and the steam may be conveniently heated by the products of combustion before they enter the chimney shaft, the employment of superheated steam is probably more econo-

mical than steam jacketing. In the steam turbine referred to above, superheating the steam through rather less than  $40^{\circ}\text{C}$ ., so as just to prevent condensation in the working cylinder, very much increased the efficiency of the engine.

**397. Compound Engines.** When steam expands in an unjacketed cylinder, it does not avoid loss or gain of heat during the expansion, for the very hot steam entering from the boiler finds the cylinder at a lower temperature, and gives up some of its heat to the colder metal, at once introducing a dissipation of energy, and causing partial condensation if the steam be not superheated. Towards the end of the expansion the steam cools below the temperature of the cylinder, and some of the heat is restored to it, but the restoration again means the flow of heat from a hotter to a colder body, and energy is again dissipated. Some of the steam originally condensed may be re-evaporated, but this taking place at a lower temperature the steam will have lost the opportunity of doing the work it would have done if it had expanded from the high temperature and pressure. Hence, the great variation of temperature which takes place in steam which undergoes a high rate of expansion causes an alternating flow of heat to and from the walls of the cylinder, and this involves dissipation of energy. To overcome this difficulty it is necessary to reduce the range of temperature which obtains within the cylinder, and this is effected in the compound engine by allowing the expansion to take place in two stages and in two separate cylinders. The steam, for instance, may expand to  $2\frac{1}{2}$  times its original volume in the high-pressure cylinder, the expansion being accompanied by a corresponding fall of temperature, the extent of which will not be very great. It is then admitted into the low-pressure cylinder, which is always at a lower temperature than the other, and may there again expand to  $2\frac{1}{2}$  times its volume, giving a total expansion to  $6\frac{1}{4}$  times the original volume. Where very high ratios of expansion take place, as in modern marine engines, three cylinders are employed in succession, forming "triple expansion"

engines, so as still further to sub-divide the range of temperature, and in some cases quadruplex expansion is adopted. With such engines steam may be expanded to thirty times its original volume; but if this expansion took place in a single cylinder, the capacity of the cylinder would have to be sufficient to allow of the full expansion, while its strength would have to be proof against the full boiler pressure of the steam, thus necessitating an enormous mass of metal, in which the steam would be exposed to a very wide range of temperature, and the consequent dissipation of heat would be very great. In the steam turbine, in which the steam expands to 150 times its original volume, the high-pressure steam is always at one end of the cylinder or drum, and the low-pressure steam at the other end; so that the same metal is always in contact with steam at the same temperature, and the only losses are due to the conduction of heat through the substance of the metal from the high-pressure to the low-pressure portions.

398. The condensation of steam in the cylinder in consequence of the conversion of heat into work is, of itself, by no means an evil, and if the condensed steam would leave the cylinder as *water*, the condensation would improve the efficiency of the engine; but during the exhaust the condensed steam is again evaporated, and the latent heat of evaporation is taken from the cylinder, cooling the latter and leading to a partial condensation of the high-pressure steam during the next period of admission, unless the cylinder is jacketed. The portion of the steam which condenses during expansion gives up its latent heat to the contents of the cylinder, thus tending to keep up the temperature, and with it the pressure, of the remaining steam. The condensation of a portion of the steam causes, of course, a falling off of pressure, but the giving up of the latent heat increases the pressure more than three times as much as the loss of steam diminishes it, and hence the pressure does not fall nearly so rapidly when saturated steam expands as when superheated steam or air expands. The partial condensation would therefore be beneficial but for the re-evaporation and

consequent loss of heat mentioned above. This point will be made more clear when we speak of indicator diagrams.

399. **Superheating.** Sometimes it is convenient, however, to employ steam which is not saturated when it enters the cylinder, but superheated. The steam in the boiler being in the presence of water is of course saturated, but on leaving the boiler it is made to pass through heated pipes, whereby its temperature is raised without, of course, affecting its pressure, and the steam becomes *superheated*. The steam thus superheated can afford to lose a certain amount of heat without condensing, and if sufficient heat has been communicated to it, it may be capable of expanding to many times its volume, doing work all the while, and still remain dry steam. In engines of the locomotive type this superheating is generally effected by causing the steam on leaving the boiler to pass through tubes placed in the smoke box of the engine, and exposed to the action of the products of combustion as soon as they have passed through the boiler tubes. With the high pressures now employed it is impossible to superheat the steam to any great extent without injuring the cylinders and packing. In order that the steam may be as dry as possible when it leaves the boiler, the steam-pipe collects the steam from a steam chest placed above the general level of the crown of the boiler.

In starting a locomotive the valves are generally so arranged that for the first few strokes the cylinder is filled with steam at full boiler pressure. This serves the double purpose of raising the temperature of the cylinder as quickly as possible, and also of keeping up the pressure on the piston just when the greatest pressure is required, viz., in starting the train.

400. When steam is employed expansively, it is seldom that its temperature remains even approximately constant during the expansion, though it is very usual to assume that this is the case in rough calculations of the work that may be expected from a stroke of an engine. On the other hand, no engine works so rapidly that there is no interchange of heat between the steam and the



cylinder. The conditions under which steam expands in the cylinder of an engine are generally intermediate between those corresponding to constant temperature and those which obtain when no heat enters or leaves the steam.

401. The advantages to be derived from compounding an engine depend very much upon the actual sizes of the cylinders and the rapidity of the strokes, and hence experiments made on small laboratory engines are apt to be very misleading if the conclusions derived from them are applied in practice. In a cylinder 5 inches in diameter no portion of the steam is more than  $2\frac{1}{2}$  inches from the walls of the cylinder, and the *average* distance of the steam from the metal is less than an inch. Hence it is easy for an interchange of heat to take place rapidly between the steam and the iron; but in a cylinder 90 inches in diameter the conditions are very different. It is difficult to suppose that much interchange of heat can take place between the volume of steam filling such a cylinder and the walls of the cylinder itself during the half-second occupied in the stroke of a large marine engine. Hence, as the size of the cylinder increases, the importance of compounding for the purpose of preventing interchange of heat between the metal and the steam diminishes, and the importance of jacketing diminishes also. This subject will be referred to again in Art. 435.

402. **Design of Boilers.** In designing a boiler, the first question to be determined is, of course, the amount of steam required per hour and the pressure at which it is to be generated. The type of boiler will be generally determined by local circumstances. For example, for marine and locomotive boilers reduction of weight is of very great importance; and in marine boilers especially, high efficiency is essential, as it is necessary to consider not simply the cost of the coal consumed, but the space occupied by the coal to the exclusion of remunerative cargo. For stationary boilers the chief consideration is efficiency and durability, the cost of repairs in boilers of the locomotive type being generally far greater than in the case of Lancashire boilers. Suppose that the boiler

is required to generate 3,600 lbs. of steam per hour at a pressure of 160 lbs. per square inch, and is to be of the Lancashire type, the condensed steam being returned to the boiler at a temperature of  $50^{\circ}\text{C}$ . Under these circumstances, one pound of good steam coal will, with careful firing, and provided that the boiler is sufficiently large to generate the steam without being "pressed," produce about 9 lbs. of steam. It will therefore be necessary to provide for the consumption of 400 lbs. of coal per hour, and as it is undesirable to attempt to burn more than 20 lbs. of coal per square foot of grate surface per hour, this implies about 20 square feet of grate. Two flues, each 30 inches in diameter, with fire bars measuring 5 feet from back to front, will provide rather more than this amount of grate area. The distance from the fire door to the bridge will then be nearly 6 feet, which is as long a furnace as is consistent with easy firing. With hand firing, in Lancashire boilers, the coal consumed per hour is not generally more than 15 lbs. per square foot of grate area. A thirty-inch flue will enable the width of the fire bars to be about 2 ft. 3 in., and two such flues will necessitate a diameter for the boiler shell of about 7 feet. The circumference of the boiler will be about 22 feet, and of this about 12 feet will be exposed to the action of the furnace gases. The circumference of the two flues together will be nearly 16 feet, making 28 square feet of heating surface for every foot in the length of the boiler. The area of the flues below the fire bars, that is of the ash pits, is not reckoned as part of the heating surface; but this may be more than compensated by the introduction of four or five "Galloway tubes" into each flue. If the length of the boiler be 26 feet, there will be a total heating surface of about 728 square feet, or nearly 36 square feet of heating surface for each square foot of grate. The production of 3,600 lbs. of steam per hour will then represent only 5 lbs. of steam per square foot of heating surface, and this can be readily maintained or exceeded.

403. In an experiment conducted by the writer upon a Lancashire boiler 7 feet 6 inches in diameter and 30 feet long, with natural draught produced by a chimney

120 feet high and 6 feet in diameter internally (the chimney was designed for a range of 5 boilers), it was found possible with best locomotive steam coal to produce about 7,000 pounds of steam per hour with cold feed-water. This boiler possessed about 36 square feet of grate area, and each flue was crossed by Galloway tubes. This may be taken as the limit of the capacity of such a boiler with ordinary chimney draught. The boiler was guaranteed by the makers to be capable of producing 6,000 lbs. of steam per hour. The coal consumed amounted to about 24 lbs. per hour per square foot of grate area. This was only possible with large coal of excellent quality.

**404. Chimney Design.** The amount of air requisite for the combustion of 400 lbs. of coal may be taken to be about 6,000 lbs. This will represent about 78,000 cubic feet of air at ordinary temperature, or the air supplied to the boiler must be about 22 cubic feet per second. If the temperature of the chimney be as low as  $360^{\circ}\text{C}$ ., this will correspond to about 48 cubic feet of gases passing up the chimney per second. A velocity of 10 feet per second is a reasonable velocity for a chimney draught, which should never much exceed 16 feet per second. This gives a sectional area of 4.8 square feet, corresponding to a diameter of nearly 30 inches. For large chimneys, 100 feet in height, one square foot of sectional area for each hundredweight of coal consumed per hour will generally suffice.

In chimneys which are narrowed at the top, so as to give a great velocity to the issuing gases, a considerable amount of energy is carried away by the stream, so that an appreciable portion of the water-gauge produced by the chimney is employed in giving the necessary velocity to the issuing gases. Thus, if the absolute temperature of the gases is twice that of the external air, and the chimney is so narrow that the velocity of the issuing stream is 32 feet per second, the result will be that 16 feet of the chimney's height will be wasted in communicating this velocity to the gases instead of being effective in drawing the air through the fires and flues of the boiler.

405. If the diameter of a chimney is altered, while the quantity of gases passing through it remains the same, the water-gauge required to overcome the friction of the chimney varies approximately inversely as the fifth power of the diameter. If therefore a chimney is so designed that the water-gauge required to overcome the friction is one-fiftieth of an inch, which will be equivalent to that due to about three feet of the chimney, a diminution of diameter to one-half will raise the water-gauge necessary to overcome the friction to about  $\cdot 64$  inch, or the whole amount due to the chimney if its height is about 90 feet. It is obvious that such a chimney would be useless for the purpose; while, on the other hand, it is clear that when the resistance has been reduced so as to require a water-gauge of only  $\cdot 02$  inch to overcome it, there can be little object in increasing the diameter in order further to diminish the friction.

406. The friction of any horizontal flue through which a steady current of gas is passing may be readily determined by connecting a pressure-gauge of sufficient delicacy to each end of the flue. The difference between the indications of the gauges gives the pressure necessary to overcome the friction, provided that the velocity of the gas is the same at each end. When the flue is not horizontal, a correction must be made for the difference between the pressure due to the column of gas in the flue and a corresponding column of atmospheric air. The friction of a chimney may be determined by observing the composition and temperature of the gases within the chimney at various points of its height, and thus calculating the theoretical water-gauge of the chimney. The difference between this and the indication of a water-gauge communicating with the interior of the chimney at its base measures the pressure required to overcome the friction of the shaft.

407. When the temperature of the furnace gases in a chimney is raised, the pressure which forces the air through the flues will be increased in proportion to the difference between the weight of the contents of the chimney and that of an equal volume of the external air. But if the

same quantity of air passes through the fire, the velocity of the gases in the chimney varies inversely as their density, and, therefore, directly as their absolute temperature. But the frictional resistance of the chimney, and the loss of pressure due to the velocity of the escaping gases, both vary,—the one approximately, the other exactly, as the square of the velocity and as the density conjointly. Hence, to maintain the same draught through the chimney the pressure would have to vary inversely as the density of the gases, and, therefore, directly as the absolute temperature. If the whole of the resistance were due to the chimney friction, and to the velocity of the escaping gases, the draught would be a maximum when the absolute temperature of the gases in the chimney was twice that of the external air, corresponding in general to a temperature of about  $300^{\circ}\text{C}.$ ; but since a great portion of the resistance is due to the air passing between the fire bars and through the fires, where it is unaffected by the temperature of the chimney, the draught may be increased, though very slightly, by raising the temperature of the chimney gases somewhat above  $300^{\circ}\text{C}.$  In a well-designed chimney, 100 feet in height, the pressure required to overcome the friction of the chimney itself will not generally exceed one and a half times that necessary to produce the actual velocity in the gases. If the velocity be restricted to 16 feet per second, the density being half that of the external air, the loss of power due to the velocity of the escaping gases will be equal to that due to 4 feet of the height of the chimney, and the loss due to the friction of the chimney will not exceed that due to six feet of height; so that ninety feet of the height of the chimney will be efficient in producing the draught through the boiler.

408. **Efficiency of Boilers.** The efficiency of a boiler can be readily tested by a careful examination of the state of the gases in the smoke-box, uptake, or chimney base. If the boiler is properly covered by a suitable non-conducting composition (cork fragments, silicate wool, etc.), the loss of heat by radiation from its surface cannot be considerable. The only escape for the heat gener-

ated in the furnace will then be through the steam or with the gases up the chimney. There will therefore be only two points requiring attention, viz.: (1) That all the heat is actually generated which the coal is capable of producing, in other words, that the combustion is perfect; and (2) that as little of the heat so generated escapes up the chimney as is consistent with the maintenance of an efficient draught.

409. Both these conditions can be tested by examination of the temperature and composition of the gases in the uptake. To secure the first condition it is only necessary that there should be no smoke and no combustible gases remaining, in particular that there should be no carbonic oxide. Reference to Art. 64 will show the importance of this. The second condition will be secured if the temperature of the furnace gases is as low as possible consistent with efficient draught ( $300^{\circ}$  C. may be regarded as a satisfactory temperature, but a lower temperature may be secured), and if the amount of air supplied to the fires is as small as possible consistent with complete combustion. Unfortunately it is impossible to utilize the whole of the oxygen in the air, and hence a considerable surplus has to be employed; so that 15 lbs. of air or more, instead of 12 lbs., will generally be required to secure the complete combustion of each pound of coal. If the boiler is working much below its full power, so that the consumption of coal per square foot of grate per hour is very small,—say considerably below 10 lbs.,—the amount of air passing through the fire will generally be excessive; and if any portion of the fire bars be left uncovered, through careless firing, the same result will ensue. Sometimes the amount of air entering the fires per pound of coal consumed exceeds 30 lbs., and thus the loss of heat through the furnace gases is more than doubled. In the case of a boiler with brick flues, any crack in the flue walls allowing air to enter produces a similar result. If the composition of the coal and of the ash is known, the analysis of the furnace gases will at once indicate the quantity of air entering the flues per pound of coal consumed. The carbon present, as carbonic anhydride, or

in other forms, determines the amount of coal, and the free oxygen the surplus of air. Roughly speaking, 10 per cent. of free oxygen indicates that the amount of air supplied is twice that required for the complete combustion of the coal. Five per cent. of free oxygen indicates that the air is one-third more than that required for combustion; and so on.

410. If the amount of free oxygen in the furnace gases is less than five per cent., it is clear that the air supplied is not much in excess of the minimum consistent with perfect combustion; if no combustible gases are present and no smoke, it is clear that the combustion is perfect, and that all the heat which the coal is capable of producing is generated; and if the temperature of the furnace gases on entering the chimney does not exceed  $300^{\circ}\text{C}$ ., the heat lost by this means is almost as small as possible, and the rest must be utilized in the production of steam after allowing a small amount for loss by radiation and conduction. If two boilers give the same results as far as these tests are concerned, and make the same amount of ash, or other unconsumable residue from the fuel, they *must* produce the same amount of steam per pound of coal, and direct experiments for the comparison of their evaporative efficiency become unnecessary, however much the boilers may differ in construction. Of course the lowest temperature which it is possible for the furnace gases to possess is that of the contents of the boiler, but to cool them within a few degrees of this temperature requires an enormous amount of heating surface. With 16 lbs. of air per pound of coal consumed, and a chimney temperature of  $300^{\circ}\text{C}$ ., the loss of heat in the furnace gases is about 15 per cent. of the total heat generated. The most economical plan for utilizing the excess of heat contained in the furnace gases is to use them to heat the feed-water by means of an "Economiser," which consists of a stack of cast-iron pipe placed in the main flue near the chimney base, through which the feed-water is made to flow. The furnace gases thus, just before entering the chimney, come in contact with pipes containing comparatively cold water, and readily

give up a large portion of their heat to be conveyed to the boiler with the feed-water.

411. A rough calculation of the loss of heat by radiation from an uncovered boiler has been given in Art. 317. This may be reduced to one-eighth, or even less, by the employment of a suitable covering for the boiler, in which case the loss of heat by radiation becomes so small that it need scarcely enter into the calculation of the coal consumption.

It should be noticed that, while with boilers of the same type the ratio of the total heating surface to the grate area gives a fair idea of the efficiency of the boiler, this test cannot be extended to boilers of different types, as very much depends on the situation of the heating surface relative to the fire, and to the direction of the flow of the gases. If the gases impinge directly upon the surface, it is much more efficient than the same area of a tube through which the gases pass longitudinally, while the lower portions of the surfaces of tubes through which the furnace gases pass generally become covered with soot and coal dust, carried forward with the draught, and are of little value as heating surface.

412. **Refuse Destruction.** The production of steam for useful purposes by the waste heat from the destruction of ash-bin refuse is a subject which has lately received a considerable amount of attention. Where the purposes for which the steam is required are such that the steam may be generated at a nearly uniform rate through the whole twenty-four hours of the day, the problem presents no special difficulty. It is only necessary to secure that the distillate from the refuse shall be completely consumed before any of the furnace gases are cooled by contact with the boiler plates or tubes; otherwise white fumes consisting of combustible material appear at the top of the chimney shaft, and can be traced for great distance through the air. This complete combustion may perhaps be best secured by building a considerable number (five or six at least) of furnaces in one range, communicating with a common flue of ample capacity.



Sometimes the furnaces are built each with its own fire-brick combustion chamber above the crown of the furnace, in which several perforations are made by leaving out bricks, so that the furnace gases get ready access to the combustion chamber, whence they pass into the common flue. The chimney draught is artificially increased by means of a fan, or by steam jets introduced below the fire bars. The temperature in the combustion chambers of the several furnaces will depend on the stage of combustion of the charge of refuse on the bars. When the combustion has reached its most vigorous stage, the combustion chamber is heated to very bright redness, and the temperature probably exceeds  $2000^{\circ}\text{F}$ . In the common flue, which is also lined with refractory material, and, like the furnaces and combustion chambers, enclosed in such a mass of brickwork as to render the loss of heat by conduction and radiation comparatively small, the intensely hot gases from those furnaces in which the charges are in a state of vigorous combustion mingle with the cooler gases or distillates from furnaces which have been recently charged, and raising their temperature, secure their complete combustion. The high temperature of the brick lining of the combustion chamber, due to the previous charge, goes far towards promoting the combustion of the distillate from a fresh charge; but any material which escapes unconsumed from the combustion chamber is completely burned up on mixing with the other gases in the common flue. On reaching the end of this flue the gases enter a large chamber, whence they pass through the tubes of the boilers. On reaching the boilers, it is found that the temperature of the gases is about  $1,500^{\circ}\text{F}$ . If the boilers are properly designed, and are producing steam at a pressure of 150 lbs. per square inch, the gases may be cooled to about  $500^{\circ}\text{F}$ . in passing through the tubes, thus giving to the water about two-thirds of the heat derived from the combustion which they have brought with them. To secure complete combustion a considerable excess of air is introduced by the forced draught, and this causes the temperature, to which the gases are raised, to be much lower than it would be if the quantity of air could be kept only slightly

in excess of that required for combustion ; but if no heat were lost to the brickwork, the temperature would still considerably exceed  $1,500^{\circ}\text{F.}$ , and would probably be not less than  $2,000^{\circ}\text{F.}$  If this be so, it follows that about one-quarter of the heat produced is lost through the great surface exposed by the brickwork, one-quarter escapes up the chimney shaft, and one-half is available for steam generation. To produce the same amount of heat as a ton of steam coal, the quantity of refuse required will vary with the locality from which it is collected. Speaking generally, six or seven tons of refuse may be taken as equivalent to one ton of steam coal in actual calorific power. Hence, for steam-raising purposes about ten tons of refuse will be required to produce the same amount of steam as one ton of steam coal. When it is remembered that the ton of coal may be burned in small iron furnaces inside a boiler in two or three hours, while to burn ten tons of refuse in the same time ten or a dozen large brick furnaces with their combustion chambers and flues will be required; it is not surprising that the loss of heat by radiation and conduction is much greater in the case of the refuse than in that of the coal.

413. When the steam is not required at a uniform rate, but the demand is very irregular, as in the case of electric-lighting, where more steam is required during a few hours in the evening than during all the rest of the day, it is necessary to adopt some method of storing the energy, since the furnace when once heated must be supplied with fuel at a fairly uniform rate. If the fires are extinguished in a bank of furnaces, it requires about twelve hours to get them into steady action again. It is therefore impossible to treat the destructors as ordinary boiler fires. Moreover an enormous number of furnaces would be required to supply steam during the hours of maximum load of a large electric-lighting station. The employment of storage batteries is one solution of the difficulty ; but it is expensive, and does not come within the province of this treatise. The utilisation of the storage capacity of water under high pressure is another solution which has recently been proposed. In

Art. 213 the great volume of steam produced in the event of the explosion of a high-pressure boiler was alluded to. This power of "super-heated" water to produce steam is used in the system of "thermal storage." A number of tanks, thoroughly well lagged to prevent loss of heat from the surface, are constructed like boiler shells, preferably in the form of vertical cylinders, so as to safely sustain a pressure of, say, 200 lbs. on the square inch. The boiler is constructed to produce steam at this pressure, or somewhat higher, and the tanks having been nearly filled with water, the boiler steam is blown into them so as either to mingle with the water, being condensed by it, or to pass through a coil of pipe immersed in the water, and after condensation to be returned to the boiler. By this process the steam is condensed, and the water in the tanks is heated until its temperature and the pressure of the steam above it are only just below the temperature and steam pressure in the boiler itself. Nearly all the lime contained in the water will be deposited as a soft sludge in the tanks, and will be harmless, since the tanks are nowhere exposed to fire. The feed-water for the boiler is supplied from the tanks, and thus the boiler is fed with water which has been "purified" in the best possible way. When the reserve of energy is required to be drawn upon, the tanks are connected with the engines, and will continue to supply them with steam until the temperature in the tanks has fallen to the lowest point at which steam can be generated at sufficient pressure for the engine. If, for example, the engines require steam at a pressure of 100 lbs. per square inch, which means a temperature of  $338^{\circ}\text{F.}$ , and the tanks are charged up to a pressure of 200 lbs. on the square inch, which corresponds to  $388^{\circ}\text{F.}$ , then the tanks will be able to supply the engine with steam until their contents have fallen in temperature to  $338^{\circ}\text{F.}$  If no heat were lost by the tanks, and if operations were commenced with the tanks at  $338^{\circ}\text{F.}$ , and saturated steam from the boiler were blown into the tanks at the pressure existing in them until a temperature of  $388^{\circ}\text{F.}$  and pressure of 200 lbs. per square inch were reached, then it is obvious that in allowing the temperature to fall again to  $338^{\circ}\text{F.}$  the

tanks would give up exactly as much steam as they received.

414. The total heat of steam at  $T^{\circ}\text{F.}$  is, in Fahrenheit units,  $1,091\cdot7 + \cdot305 (T - 32)$  or  $1,081\cdot94 + \cdot305 T$ . Hence to produce from ice-cold water a pound of steam at  $338^{\circ}\text{F.}$ , at a pressure of 100 lbs. per square inch,  $1185\cdot03$  units of heat are required, of which  $388 - 32$  or  $356$  units, are already contained in the water at  $388^{\circ}\text{F.}$  Hence  $829\cdot03$  units must be derived from the water left behind, which is cooled through  $50^{\circ}\text{F.}$ , and therefore  $16\cdot58$  lbs. of water must be left behind; so that, for every pound of steam produced, there must be originally in the tanks  $17\cdot58$  lbs. of water at  $200^{\circ}\text{F.}$  As the steam at first will be produced at a higher temperature,—and if it is to enter the engine at a pressure of only 100 lbs. per square inch, the pressure will have to be reduced by throttling (accompanied by super-heating),—the steam will carry with it a little more heat than has been calculated above, and nearly 18 lbs. of water will be required to produce 1 lb. of steam.

If the boiler generates 3,000 lbs. of steam per hour for twenty hours, and the power is required for the remaining four hours at the average rate of 18,000 lbs. of steam per hour, of which 3,000 can be obtained directly from the boiler and 15,000 from the reservoirs, we shall require a storage capacity of  $20 \times 3,000 \times 18$  pounds of water; that is, of 108,000 gallons. This corresponds to about fourteen cylindrical tanks each 8 feet in diameter, and filled with water to a height of about twenty-five feet.

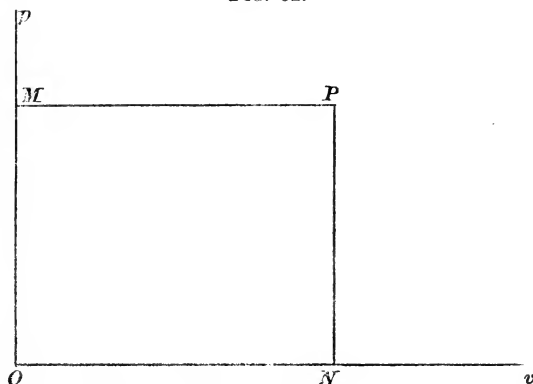
NOTE.—Joule's value of the Mechanical Equivalent of Heat has been adhered to in all the illustrations given, and the value of  $J$  has been taken to be 1390 foot-pounds (or 772 foot-pounds if Fahrenheit's scale be used). The researches of Rowland, published in 1883, however, showed that this value is too small, the difference being due mainly to errors in Joule's thermometer, but partly to an erroneous value having been assigned to the specific heat of water at the temperature at which Joule worked. Recent researches of a most elaborate character, carried out at Cambridge by Mr. E. H. Griffiths, confirm Rowland's conclusion, and show that the Mechanical Equivalent of heat, in round numbers, may be taken as 1400 foot-pounds with reference to the Centigrade degree, or 778 foot-pounds for the degree Fahrenheit.

## CHAPTER XII.

### ON INDICATOR DIAGRAMS, BOILERS, ETC.

415. **The Indicator Diagram.** If we draw two straight lines  $Ov$  and  $Op$  at right angles to one another, and agree that distances measured from  $O$  along  $Ov$  shall

FIG. 52.



represent volumes; while distances measured along  $Op$  represent pressures, then any point,  $P$ , may be regarded as representing the condition of a substance, as far as its volume and pressure are concerned; for drawing  $PN$  and  $PM$  perpendicular to  $Ov$  and  $Op$  respectively,  $ON$  or  $MP$  may be taken as representing the volume of the substance, and  $OM$  or  $NP$  its pressure.

416. If the volume and pressure of the substance undergo any variations, but are always represented by the point  $P$ , it is clear that  $P$  must change its position, and since the condition of a substance cannot change

except by a steady increase or decrease (although this may take place very rapidly), it follows that if the point  $P$  always represent both the volume and pressure,  $P$  must move along some definite line, generally curved; and this line will, by its form, represent graphically the law connecting the variations in volume and pressure which the substance undergoes. Such a line is called an indicator diagram.

417. **Isothermal Lines.** For example, suppose the substance to be a gas which is kept at constant temperature and strictly obeys Boyle's law. Then, if the pressure be doubled, the volume will be reduced to one-half, and if the volume be doubled, the pressure will be correspondingly reduced, the product of the volume and pressure remaining always constant. Hence,  $P$  will move so that the rectangle  $PMON$  is of constant area. (This is a condition fulfilled by a rectangular hyperbola, having  $Ov$  and  $Op$  for the asymptotes, so that the rectangular hyperbola is the graphic representation of Boyle's law.) As the volume increases indefinitely, the pressure will diminish indefinitely, and the point  $P$  will approach the line  $Ov$ , but will never reach it. Similarly, if the pressure be increased,  $P$  will approach the line  $Op$ , but never reach it. The curve thus traced represents the relation between pressure and volume for a given mass of air kept at constant temperature, and is called an *isothermal* line.

418. If the substance does not obey Boyle's law, the isothermal line will differ from that above described. For example, in the case of carbonic anhydride at, say  $15^{\circ}\text{C.}$ , the product of the pressure and volume will be very nearly constant if the pressure is less than two or three atmospheres, and the isothermal line will nearly coincide with the corresponding (hyperbolic) line for air; but at higher pressures the volume is less than Boyle's law indicates, and the isothermal curve for carbonic anhydride falls below the air line. When, at length, the pressure is reached at which liquefaction commences, the pressure remains constant, while the volume decreases until the whole mass is liquefied, and the isothermal line consequently becomes horizontal. As soon as the whole is

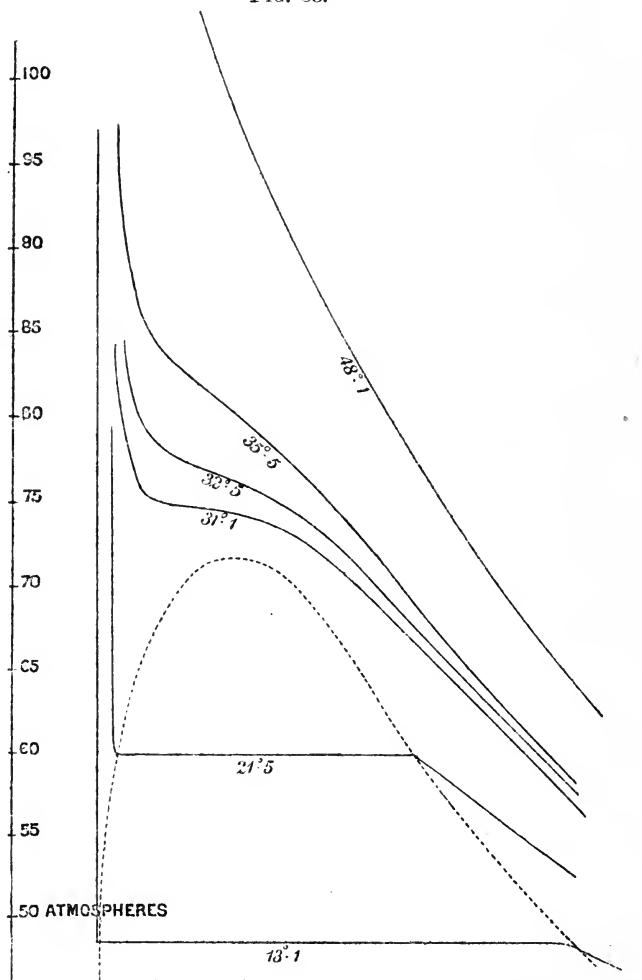
liquefied, the diminution of volume with further increase of pressure is comparatively slow, and the line slopes upwards towards *Op* at a great and increasing inclination.

419. When a quantity of vapour is saturated, if the temperature is kept constant, any diminution of volume produces condensation, and no increase of pressure can take place until the whole has been liquefied. Hence, the isothermal line of a substance which is partly liquid and partly a saturated vapour is necessarily horizontal. When the whole of the substance has been liquefied, the pressure may have to be increased enormously to produce any sensible diminution of volume, and the isothermal line becomes nearly parallel to *Op*. This is the case with water at ordinary temperatures, but there are some liquids which, like liquid carbonic anhydride, are extremely compressible when on the point of boiling, and the isothermal line for such liquids is at first considerably curved; but as the pressure is increased, the compressibility diminishes and the line becomes more nearly parallel to *Op*.

420. **Critical Point of Carbonic Anhydride.** The accompanying figure shows the isothermals for carbonic anhydride corresponding to the temperatures indicated. The dotted line is drawn through all the points at which liquefaction commences and terminates, so that all points which correspond to the existence of the substance partly as liquid and partly as vapour lie below the dotted line. It will be noticed that at  $13.1^{\circ}\text{C}$ . there is a considerable difference between the volume of the substance as vapour (or gas) and its volume as liquid; hence the horizontal line, within the dotted curve, corresponding to a state partly liquid and partly gaseous, is of considerable length. At  $21.5^{\circ}\text{C}$ . the volume of the liquid is much greater, while the volume of the gas when liquefaction is about to commence is much less; so that the portion of the isothermal within the dotted curve is much less, and it will be noticed that it differs slightly from a horizontal straight line. At higher temperatures the difference between the liquid and gaseous states is still less, and the isothermal of  $30.92^{\circ}\text{C}$ . (not shown in the figure) just *touches* the top of the dotted curve without

cutting it. This point of contact corresponds to the critical point. The isothermals above this do not meet

FIG. 53.



the dotted curve at all, and there is no range over which the substance is partly liquid and partly gaseous. In



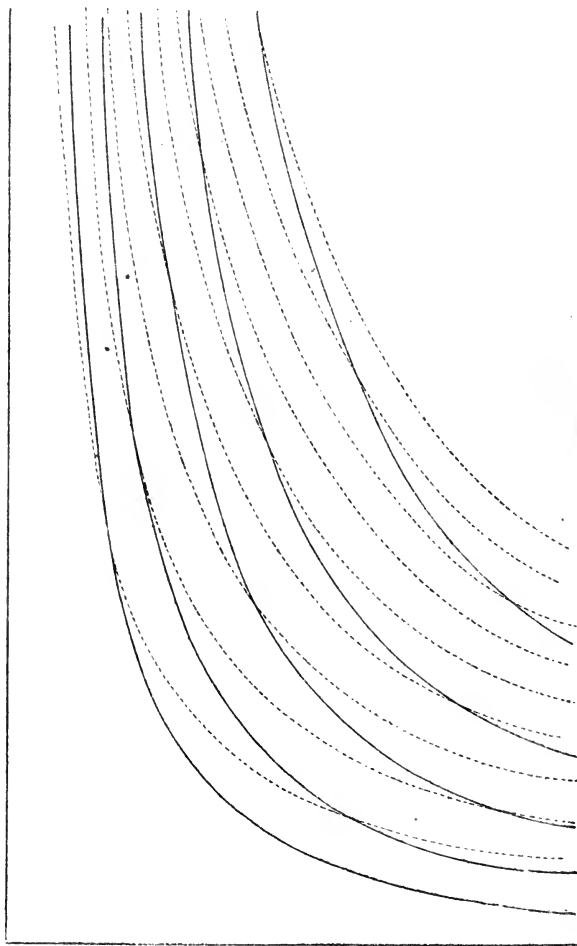
fact, as previously stated, at temperatures above the critical point no amount of pressure will liquefy a gas; and then, and only then, is it strictly entitled to be called a gas. When in the gaseous state at a temperature below the critical point, it should be called a vapour. It will be noticed that the isothermals for some distance above the critical point still show a tendency to contrary flexure just above the dotted curve, as though the substance were still inclined to liquefy, and the departure from Boyle's law is very great. At  $48.1^{\circ}\text{C}$ ., however, this tendency has quite disappeared, and the isothermal is more like that of a perfect gas. For carbonic anhydride the pressure at which condensation commences at  $13.1^{\circ}\text{C}$ . is about 47 atmospheres; at  $21.5^{\circ}$  it is about 60 atmospheres; while the critical pressure is nearly 75 atmospheres.

**421. Isentropic Lines.** If a quantity of gas be compressed while no heat is allowed to enter or leave it, its temperature will be raised on account of the compression, and the pressure will consequently be increased more than if the temperature had been kept constant. Hence, if we represent the several conditions through which the substance passes by a line on the indicator diagram, such a line will at every point be inclined at a greater angle to the line of volumes than the corresponding isothermal, and hence will cut across the successive isothermals (as the temperature of the substance is raised). A line representing the relation between the pressure and volume of a substance when no heat is allowed to enter or leave it, is sometimes called an *adiabatic* line, but the term *isentropic* line is preferable. In the accompanying figure, the dotted lines represent isothermals for a perfect gas, and the continuous lines isentropics. (Fig. 54.) The relation between the pressure and volume of a perfect gas when no heat enters or leaves it is  $p v^{\gamma} = \text{a constant}$ , where  $p$  represents the pressure,  $v$  the volume, and  $\gamma$  the ratio of the specific heat at constant pressure to the specific heat at constant volume, or about 1.42 in the case of most gases.

**422. Watt's Indicator.** The indicator diagram was first employed by James Watt to indicate the pressure and

volume of steam in the cylinder of an engine (on one side of the piston) at every point throughout the stroke. In

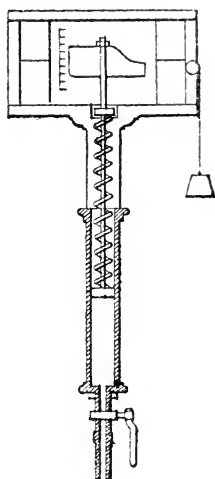
FIG. 54. Art. 427.



Watt's indicator the pressure was registered by a pencil which was attached to a piston moving in a very small

cylinder to which the steam from the engine cylinder had access. The piston was pressed down by a spring, while the steam gaining access to its lower side forced it up against the pressure of the spring. It is well known that in helical springs the compression or extension is proportional to the force applied; all spring balances are based on this principle: hence, the pencil of the indicator was raised through a distance proportional to the excess of the pressure of steam in the cylinder over the atmospheric pressure; or if the steam pressure at any time were less than atmospheric, the pencil was depressed through a corresponding distance. As it moved the pencil traced a line upon a sheet of paper attached to a board which could be made to move in exactly the same way as the piston (though the length of the traverse was reduced). If the pressure in the cylinder remained constant, a horizontal line would be traced by the pencil on the moving board; if the pressure increased or diminished when the board was stationary, or changed *suddenly* while the board was moving, a vertical line would be the result. A steady change of pressure accompanying the change of position of the piston would cause a curved line to be described.

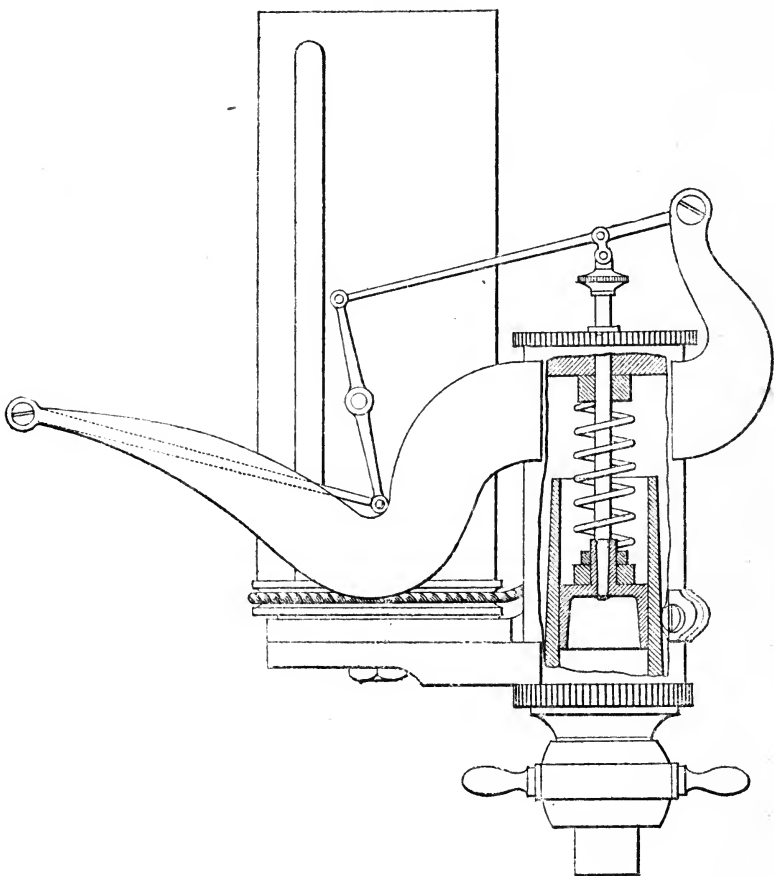
FIG. 55.



423. It was found, however, that, although the traverse of the pencil in Watt's indicator was only a few inches, when the pressure varied rapidly the kinetic energy acquired by the little piston, pencil, etc., was sufficient to carry the piston and pencil far beyond the position corresponding to the steam pressure. This difficulty was obviated by allowing the piston to move only through a very small distance, correspondingly strong springs being employed, and the motion of the piston was multiplied in the pencil by a system of levers. The accompanying figure shows Richards' indicator, in which this multipli-

cation is effected by a combination of levers similar to Watt's "parallel motion." In Dark's high speed indicator the pencil moves in a vertical guide, the lever actuating

FIG. 56.



it having a longitudinal slot cut in it to allow of this motion of the pencil. In all indicators now in use the paper is wrapped upon a cylinder, or drum, which is made to revolve through rather less than one revolution

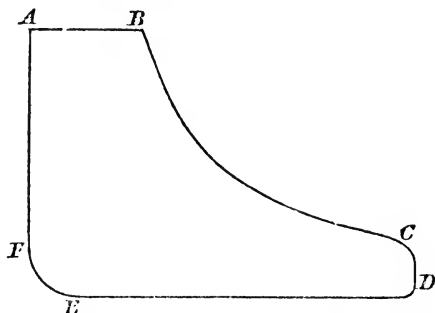
by the motion of the piston of the engine, and is made to return by a spring contained within the instrument. As the stroke of the engine is generally much longer than the longest diagram the indicator can draw, it is necessary to employ some mechanism by which the drum of the indicator may be moved through a distance less than that of the piston but always strictly proportional to it. Springs of different strengths are generally supplied with the instrument, and so arranged that a movement of one inch on the part of the pencil corresponds to a steam pressure varying from 10 lbs. to 80 lbs. on the square inch, according to the particular spring employed.

**424. Normal Indicator Diagrams.** If the steam from the boiler be allowed full access to the cylinder during the whole of the forward stroke of the piston, and the steam in the cylinder is allowed to escape freely into the air during the whole of the backward stroke, the ports being opened and closed at the instant when the piston is at the extreme end of its stroke, the diagram traced by the indicator will be a rectangle, the upper side corresponding to boiler pressure and the lower side to atmospheric pressure. The diagrams of some steam fire-engines are of this character, for in these engines the object is to get as much work as possible from the engine in a given time, and that regardless of the expenditure of fuel. Hence the full boiler pressure of the steam is sustained till the end of the stroke.

But in all engines in the working of which economy is a consideration, the "exhaust steam" is not allowed to escape at full boiler pressure, but the supply from the boiler being cut off long before the stroke is completed, the steam in the cylinder is made to expand, doing work as it expands, and is not allowed to escape until its pressure has been reduced very much below that of the steam in the boiler. The accompanying diagram may be taken as typical of a high-pressure engine working expansively. The line *AB* represents the condition of affairs during the period of admission, and the communication being open with the boiler the pressure is maintained nearly equal to that in the boiler itself. At

*B* the steam port is closed, and the steam being confined within the cylinder expands as the piston continues its motion, tracing out the curve *BC* until at the point *C* the exhaust port is opened, and the steam being allowed to escape the pressure falls rapidly almost to atmospheric pressure. At *D* the piston is at the extremity of its stroke; it then returns, the indicator tracing out the line *DE* with the exhaust port open until, on the piston arriving at *E*, the exhaust port is closed, and the small

FIG. 57.



amount of steam remaining in the cylinder is compressed so as to form an elastic cushion and assist the rebound of the piston. The compression portion of the curve is represented by *EF*, and at *F* the steam port is opened and the pressure rises immediately to nearly the full boiler pressure.

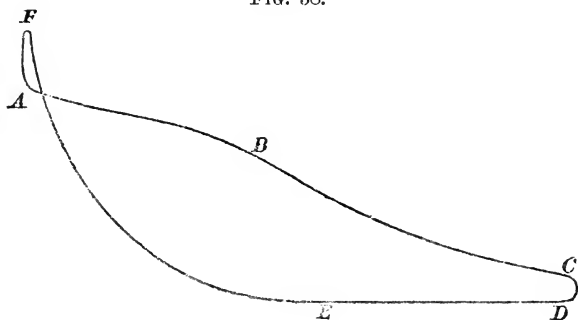
425. A glance at the indicator diagram generally shows whether the engine is working properly or whether there is anything wrong with the pistons, valves, steam ways, etc. For example, a leaky piston would be betrayed by the expansion curve falling much more rapidly than is consistent with the law of expansion of steam under the conditions obtaining in the cylinder. If the steam pipe from the boiler is too small, or if the steam port is not opened wide enough and quickly enough, when the piston commences its stroke, the supply of steam will not keep pace with the piston, and the line corresponding to the period of admission will slope downwards

from *A* to *B*. This limiting of the steam supply is called "*wire-drawing*," and the steam is said to be "*wire-drawn*." This effect also occurs in engines which are regulated by a "throttle valve" opened and closed by the governor. It always involves a waste of power, and hence it is preferable to govern by altering the position of the cut off, *B*, either by controlling the travel of the ordinary slide valve, or by a separate "expansion valve." Sometimes the wire-drawing is so marked that the period of admission can scarcely be distinguished from that of expansion.

If the exhaust port does not allow a sufficiently ready egress for the steam, the line *DE*, instead of practically coinciding with the atmospheric line, will be considerably above it. This shows that some of the work done by the steam in expanding is employed in overcoming back pressure which detracts from the work done by the engine.

426. **Actual Indicator Diagrams.** The accompanying diagram is a rough copy of one taken from an

FIG. 58.



engine employed in blowing a fan. It will be noticed that the compression commences at *E*, or at about one-half stroke, and the piston in its motion from right to left compresses the steam in the cylinder until, when the steam port is opened at *F*, the pressure is considerably greater than that of the steam in the boiler, and

the first effect of the opening of the steam port is the diminution of the pressure in the cylinder down to that existing in the boiler. As the piston advances the supply of steam is insufficient to keep up the pressure in the cylinder, and the consequent "wire-drawing" is indicated by the downward slope of the admission line *AB*. Such a diagram shows that there is a great waste of energy accompanying the working of the engine. A readjustment of the slide valve, making the compression much later and opening the steam ports earlier, so that there may be a wider aperture for the admission of the steam when the piston advances, would enable the engine to do the same amount of work with a much smaller supply of steam and, consequently, a smaller consumption of fuel.

**427. Calculation of Work Done.** If the pressure on the piston remained constant, the work done during a single stroke of the engine would, of course, be equal to the resultant pressure upon the piston multiplied by the length of the stroke. In estimating the resultant force, we must take account of the pressure on both sides of the piston. Thus, if the area of the piston is 80 square inches, the pressure on one side 60 lbs. on the square inch (*i.e.* 45 lbs. above atmospheric pressure), and that on the other side 17 lbs. on the square inch, the resultant pressure will be equivalent to 43 lbs. on the square inch, and will be equal to 3440 lbs. on the whole piston. If the length of the stroke is 2 feet, the work done during the single stroke will be 6880 foot-pounds. If similar conditions obtain during the return stroke, the whole work done for one revolution of the engine (if it has only a single cylinder) will be 13,760 foot-pounds.

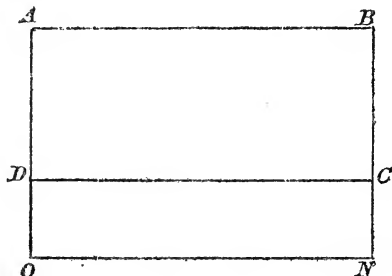
But instead of considering the pressure on both sides of the piston at once, it is generally more convenient to credit the steam on the one side of the piston with the *whole of the work* it does during the advance, and debit it with the work which has to be done against the steam pressure *on the same side* during the return of the piston. The balance is the amount of useful work obtained from the steam on that side of the piston during the complete



revolution of the engine. The steam on the other side is then treated in the same manner. It will be seen presently that this balance, or total amount of work done by the steam on one side of the piston, is shown by the indicator diagram for that side.

428. Suppose, as before, that during the whole of the forward stroke of the piston the pressure is constant; and constant, but, of course, considerably less, during the return stroke. The indicator diagram will then be a rectangle,  $ABCD$ , and upon a certain scale, the height of its top side  $AB$  above the line of zero pressure,  $ON$ , will represent the whole pressure on the piston, while its length will represent, also on a particular scale, the

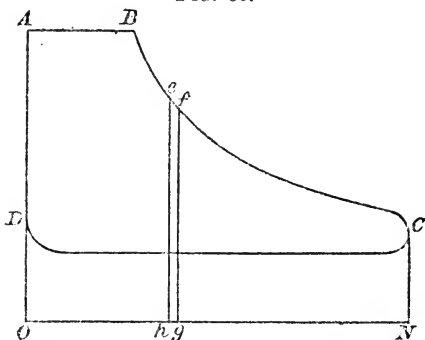
FIG. 59.



length of the stroke. Hence the rectangle  $ABNO$  will represent the work done by the steam on one side of the piston during the forward stroke. When the piston returns, the steam still exerts a pressure, represented by  $NC$  or  $OD$ , and this opposes the motion of the piston, so that an amount of work represented by  $CNOD$  has to be done *against* this pressure during the return stroke. The difference of these rectangles, viz., the rectangle  $ABCD$ , represents the useful effect of the steam on the one side of the piston during a complete revolution of the engine, and this rectangle is the indicator diagram for one end of the cylinder during the stroke. If we add to this the area of the diagram for the other side of the piston, we obtain the whole amount of work done during the double stroke.

429. Now suppose that the pressure is variable, and let the indicator diagram be  $ABCD$ . Draw  $ON$ , the line corresponding to zero pressure, and let  $CN$  touch the diagram at its extreme right. Then  $ON$  represents the length of the stroke. The work done by the steam while the piston moves over a very small portion of its stroke throughout which the pressure may, without sensible error, be regarded as constant, will be equal to the product of the pressure and the corresponding distance travelled by the piston. Thus, the work done during the portion of the stroke represented by  $ef$  will be represented by the area of the strip  $efgh$ , and so on, for other portions of the stroke. Hence the whole work done by the steam

FIG. 60.



on one side of the piston during the forward stroke will be represented by the area  $ABCNO$ , and that done *against* the steam during the return stroke by the area  $CNOD$ . The useful effect will therefore be represented by the area of the indicator diagram  $ABCD$ . In the figure of Art. 426 the area of the little loop at the top must be *subtracted from* that of the rest of the diagram in order to show the work done.

430. In employing the indicator practically for the purpose of determining the rate at which the engine is working, it is usual to use it simply for the purpose of finding the average difference throughout the stroke between the pressure of the steam at any point of the forward stroke and its pressure *at the same point* during

the return. This is represented by the average height of the indicator diagram, the average being taken by drawing a great many vertical ordinates *at equal distances* throughout the diagram, and just as the area of the diagram is equal to its average height, as thus determined, multiplied by its length (for the process is simply equivalent to dividing the diagram into strips and taking the sum of the areas of the strips), so the work done by the steam on one side of the piston during the complete revolution of the engine will be equal to the average *difference of pressure* multiplied by the area of the piston and by the length of the stroke.

Sometimes this average difference of pressure, or *mean effective pressure*, as it may be called, is found by measuring the area of the diagram with a planimeter, and dividing the area by the length. The result represents, on the scale of the diagram, the mean effective pressure. But the more usual method consists in dividing (in imagination) the diagram into a certain number (say ten) of vertical strips of equal width, and drawing and measuring the middle ordinates of these strips. The mean of these ordinates is very nearly equal to the mean height of the diagram, and represents with considerable accuracy the mean effective pressure. It is, of course, only necessary to draw these middle ordinates, and not the lines of division of the strips. For example, if ten ordinates are to be drawn, we have only to divide the length of the diagram into twenty equal parts and draw ordinates through the first, third, fifth, etc., points of division. This method clearly applies equally to condensing and to non-condensing engines, the only difference being that in the latter class the diagram lies wholly above the line of *atmospheric* pressure, while in the former the exhaust line may be very little above the line of *zero* pressure.

431. The following example will show how the rate at which an engine is working can be determined in horse-power from its indicator diagram, speed, length of stroke, and area of piston. It should be noticed that in addition to the indicator diagram it is necessary to know the area

of the piston, the length of its stroke, and the number of revolutions per minute; or instead of the two latter quantities, we may substitute the mean velocity of the piston in feet per minute.

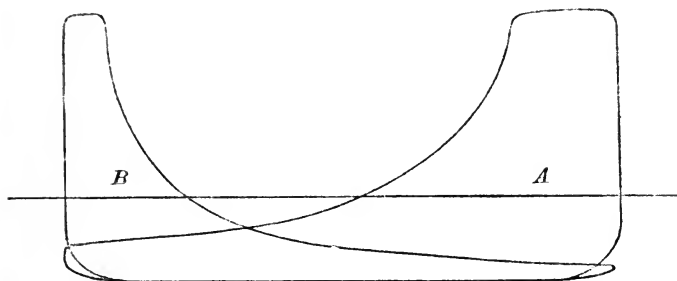
*The diameter of the piston of a single cylinder engine is 14 inches, the length of the stroke 30 inches, the number of revolutions per minute 50, the indicator diagrams the same on both sides of the piston, and the heights of the equidistant ordinates correspond respectively to  $46\frac{1}{2}$ , 50, 50,  $36\frac{1}{2}$ , 25, 18, 13, 9, 6, 4 lbs. on the square inch. It is required to find the H. P. at which the engine is working.*

The sum of the above differences of pressure is 258, and the mean effective pressure is therefore, 25·8 lbs. on the square inch. The area of the piston is  $49\pi$ , or nearly 154 square inches, and the distance travelled by the piston in a minute is 250 feet. Hence, the work done per minute is  $25\cdot8 \times 154 \times 250$ , or 993,300 foot-pounds, and the horsepower is therefore  $\frac{993300}{33000}$ , or 30·1.

432. If the diagrams obtained on the two sides of the piston are not similar, the mean effective pressure must be found for each side, and the mean of these taken. In ordinary rotary engines there is not generally much difference between the two diagrams, but inasmuch as the shortness of the connecting rod necessitates the piston being some distance in front of the middle of the cylinder in the forward stroke when the crank shaft has turned through a quarter revolution, it follows that there is usually an appreciable difference between the diagrams for the front and back of the piston. Sometimes with pumping engines nearly the whole of the work of forcing the water is done during the return or down stroke, while in the forward or up stroke there is very little load on the engine. In such a case the steam is automatically cut off as soon as sufficient has been admitted for the work of the stroke (in some engines the cut-off occurs as soon as the piston has attained a certain speed). The diagrams will then be very different, showing the difference in the amount of work done during the forward and return strokes. The accompanying figure represents two diagrams from a pumping engine belonging to the

Corporation of Nottingham. The diagram marked *A* corresponds to the top of the piston, and shows a much greater supply of steam than the diagram *B*, taken from the bottom of the cylinder. This is because the whole of the work of forcing the water to the reservoir was done in the down stroke of the engine, while in the up stroke the engine had only to raise the water from the well to the ram. By attaching a weight to the end of the beam,

FIG. 61.

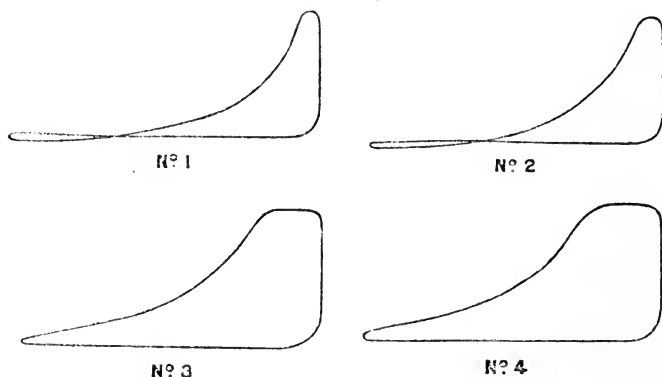


and compelling the steam to raise the weight during the up stroke, while the weight assists the steam in the down stroke, it is possible to balance the engine and make the two diagrams alike. The horizontal line in the diagram is the line of atmospheric pressure. It is usual to measure steam pressure in pounds per square inch above atmospheric pressure; below atmospheric pressure it is the common practice to measure steam pressure in inches of mercury.

433. The four accompanying diagrams show the work done by the steam on one side of the piston of a horizontal engine with a single cylinder, the governor controlling the travel of a separate expansion valve which regulated the cut-off according to the speed of the engine. The engine was making 50 revolutions per minute, the diameter of the piston was 14 inches, and the length of the stroke 30 inches. When the first diagram was taken the engine was driving only a couple of idle pulleys, and consequently had little more than its own fractional resistance to overcome. The second diagram was taken

when the engine was driving a counter-shaft and a 10-light Brush Dynamo, but no current was being generated, and the engine had therefore only the *mechanical* resistance of the system to overcome. The electric circuit was then completed, the 10 Brush lamps lighted, and the third diagram was obtained. A mortar mill and some other machinery were then connected with the engine, and the fourth diagram was the result. When

FIG. 62.



this last diagram was taken the load on the engine was rather too great for economic working.

**434. Compound Engines.** In Art. 396 reference was made to the interchange of heat between the working steam and the cylinder, and the consequent loss of power. The action will be best seen from a hypothetical indicator diagram. Thus suppose  $ABHDE$  (Fig. 63) to be the diagram corresponding to the amount of steam represented by  $AB$  on the assumption that no heat passes between the steam and the cylinder. Suppose that, on account of the action referred to, the actual amount of steam admitted to the cylinder is represented by  $AC$ , and that  $ACFDE$  is the diagram which would be drawn with this amount of steam if no condensation took place on entering the cylinder, but that an amount of steam represented by  $BC$  is at first condensed, and subsequently is gradually re-evaporated, and that the actual diagram

is  $ABGDE$ , some of the steam remaining condensed until after the exhaust has commenced. It is clear from the

FIG. 63.

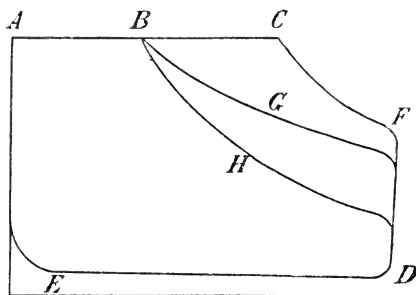


diagram that more than half the amount of work which might have been obtained from the steam corresponding to  $BC$  is wasted; and comparing the areas of the diagrams  $ABGDE$ ,  $ABHDE$ , and the corresponding quantities of steam  $AC$ ,  $AB$ , we see that the efficiency of the steam is very much diminished by the interchange of heat between the steam and the cylinder. If heat were supplied to the cylinder by a steam jacket or otherwise at a sufficient rate to prevent any cooling of the cylinder or its contents, the condensation above referred to would, of course, not take place, but the steam would be exhausted into the condenser at the full temperature of the boiler, and a considerable amount of heat would thus be carried into the condenser and wasted. The loss there produced would, however, be much less than that due to the action just described.

435. Whether it is economical to employ compound, or triple expansion engines, or engines working with simple expansion, depends on the range of pressure, and therefore of temperature, which the steam may undergo in the cylinder, and on the *speed* of the engine. The last factor is too often left out of consideration in questions relating to the relative merits of different types of engine. If an engine makes very few strokes in a minute, a long time is afforded for the interchange of heat between the cylin-

der and the steam, as the latter slowly expands; but in an engine making 600 revolutions per minute, a single stroke is completed in one-twentieth of a second, and this is all the time available for the passage of the heat into the iron and out again. In such a case the cylinder will behave almost as though it were impervious to heat, and a steam jacket will be of comparatively small benefit. In high speed engines, therefore, the boiler pressures at which it becomes desirable to employ compound, triple expansion, or quadruplex engines are much higher than in the case of slowly running engines.

436. One very important point secured by the employment of compound or triple expansion engines, is the greater uniformity of the twisting moment transmitted to the shaft. If simple expansion is employed, the stress on the piston rod is very great indeed at the beginning of the stroke, and is perhaps reduced to one-tenth of this amount before the end of the stroke. In the compound and triple expansion engine, two or three stages of expansion are going on together, and the cylinders are designed so that each piston transmits the same initial stress, and the variation of stress in each is confined within comparatively narrow limits.

In many marine engines the diameter of the low pressure cylinder especially is considerably greater than the length of stroke. In such cases the area of the cylinder covers and of the faces of the piston is great compared with that of the cylindrical surface, and it is a common practice to jacket the cylinder ends; but as these surfaces, unlike the cylindrical surface, are not exposed to wear, they might be covered with a good non-conductor of heat, and this would effectually prevent the injurious interchange of heat referred to, and probably largely increase the efficiency of the engine.

437. **Adiabatic and Saturated Curves for Steam.** The relation between the pressure and volume of wet steam when expanding without loss or gain of heat, is given approximately by the relation

$$p v^{\frac{10}{9}} = \text{a constant,}$$



and it will be seen that this differs much less from the isothermal expansion of a gas, in which  $p v$  is constant, than does the adiabatic for a perfect gas in which  $p v^{1.42}$  is constant. If, therefore, saturated steam were made to expand without receiving any heat from the cylinder, the pressure would fall at a much lower rate than if a perfect gas, or superheated steam in which no condensation took place, were expanding under the same circumstances.

If the steam were to remain just saturated, the relation between the pressure and volume would, according to Rankine, be given approximately by the equation:

$$p v^{\frac{17}{16}} = \text{a constant};$$

but this would necessitate the addition of heat to the steam. It is impossible for any steam jacket to keep the temperature of the contents of the cylinder absolutely constant, but it may be considered perfectly effective if it succeed in preventing condensation and causing the steam to expand according to the relation just given. If it were possible entirely to prevent transfer of heat between the steam and the cylinder and piston by employing a perfect non-conductor of heat, the engine would be more economical than an engine provided with steam jackets, for less heat would be carried to the condenser (on account of the exhaust steam being at a lower temperature and partially liquefied), and therefore a greater percentage would be utilized in the engine.

**438. Horse Power of Engines.** In determining the horse-power of an engine, it is important to distinguish between the indicated horse-power, or the rate at which work is done by the steam on the piston, and the brake horse-power, or power which the engine can transmit to external shafting and machinery. The difference between the two is due to the friction of the parts of the engine itself, particularly the packing rings of the piston, the packing surrounding the piston rods, the cross heads on the guides, the slide valves and eccentrics, the bearings of the crank shaft, etc. This difference may vary from ten or fifteen per cent. in large engines (of, say, 100 horse-power) and of very first-class workmanship, to

thirty or forty per cent. in small engines and engines of inferior workmanship; and in very small engines (of half a horse-power) may amount to considerably more than this. It is doubtful, however, whether 80 per cent. of the indicated power is actually transmitted to the propeller shaft in any marine engine. The amount of steam required per indicated horse-power of course depends on the boiler pressure of the steam and the pressure at which the engine exhausts. In a triple expansion condensing engine, with a boiler pressure of 160 lbs. on the square inch, a horse-power may be indicated with a consumption of only 12 lbs. of steam per hour, but 15 lbs. is considered good work. In engines capable of indicating about 100 horse-power, with a boiler pressure of 140 lbs. per square inch, and exhausting into the air, a horse-power may be indicated with a consumption of about 20 lbs. of steam per hour; and in special trials a higher efficiency than this has been obtained even with small engines.

439. The difference between the indicated and brake horse-power depends, of course, very much on the design of the engine. For example, the power absorbed by friction in a horizontal engine will generally be greater than in a vertical engine, as the weights of the pistons, cross-heads, etc., have to be supported, and necessarily produce considerable friction. Also, in the case of condensing engines, the power required to drive the air pumps and circulating pumps has to be taken from the engines, and increases the difference between the indicated power and that transmitted to the propeller shaft. Again, very large slide valves, such as are frequently employed in marine engines in order to give sufficient steam way at an early stage in the stroke, introduce a very great amount of friction, and require a correspondingly large amount of power to move them. Thus, of two engines, the first may *indicate* a considerably higher power than the second, but the power actually transmitted to the shaft may be greater in the case of the second engine.

# EXAMPLES.

## CHAPTER I.

1. DEFINE temperature. When is one temperature said to be higher than another?
2. When are bodies said to be in thermal equilibrium?
3. What method is adopted in practice to determine whether two bodies are in thermal equilibrium or not?
4. Give a definition of temperature, and explain accurately how a mercurial thermometer is filled and graduated.
5. Why should a mercurial thermometer be kept for a long while after it has been filled before it is graduated?
6. Will two thermometers, with tubes of uniform bore, filled with the same fluid, and having their freezing and boiling points accurately determined, necessarily agree at intermediate temperatures?
7. Describe some form of maximum thermometer.
8. Why is mercury used to fill ordinary thermometers?
9. What advantages or disadvantages attend the use of (1) mercury or (2) air in a thermometer?
10. A thermometer is graduated to register at the freezing-point  $15^{\circ}$  and at the boiling-point  $125^{\circ}$ . What division in it corresponds to  $104^{\circ}$  F.?
11. De Lisle's thermometer registers at the boiling-point  $0^{\circ}$  and at the freezing-point  $150^{\circ}$ . Find the temperature on the Centigrade scale which corresponds to  $95^{\circ}$  De L.
12. What temperature is denoted in Fahrenheit's scale by a number twice as large as in the Centigrade scale?

13. Two thermometers having been made from equal tubes, it is found that divisions of the same length serve as a Réaumur scale for one and as a Centigrade scale for the other. Compare the quantities of mercury in the thermometers.

## CHAPTER II.

1. EXPLAIN the difference between *temperature* and *quantity of heat*.

2. What thermal unit is generally employed to measure quantities of heat?

3. Define specific heat and describe a method by which the specific heat of a solid body may be determined. State the precautions required in practice, and the method of correcting for loss of heat.

4. Distinguish between capacity for heat and specific heat.

5. Does the specific heat of a body generally depend upon its temperature?

6. Describe an apparatus by which the quantity of heat given off by any substance in cooling from any ordinary temperature to  $0^{\circ}\text{C}$ . may be measured.

7. Describe the method of determining the specific heat of a solid by means of a Bunsen's ice calorimeter, and point out the advantages of this method.

8. Describe Regnault's method of determining the specific heat of gases under constant pressure.

9. How much coal is required to heat 40 tons of iron through  $1000^{\circ}\text{C}$ ., supposing a pound of coal in burning to produce 8000 units of heat, and the specific heat of iron to be  $\cdot 11379$ ?

10. If 60 lbs. of mercury at  $80^{\circ}\text{C}$ . are poured into 20 lbs. of water at  $10^{\circ}\text{C}$ ., what will be the temperature of the mixture?

11. A pound of water at  $200^{\circ}\text{F}$ . is mixed with a pound of mercury at  $60^{\circ}\text{F}$ . What will be the temperature of the mixture when it has become uniform, supposing that no heat is lost from it?

12. One pound of iron, at a temperature of  $100^{\circ}\text{C.}$ , is placed in 1.5 lbs. of water at  $20^{\circ}\text{C.}$ , and when the temperature of the iron and water is the same it is found to be  $25^{\circ}\text{C.}$ ; find the specific heat of iron.

13. Calculate the specific heat of a substance from the following data: 31.8 grammes heated to  $100^{\circ}\text{C.}$ , when immersed in a calorimeter containing 107 grammes of water at  $11.09^{\circ}\text{C.}$ , caused a resulting temperature of  $12.57^{\circ}\text{C.}$

14. A piece of metal, weighing 120 grammes, and at a temperature of  $100^{\circ}\text{C.}$ , is immersed in 360 grammes of water at  $19^{\circ}\text{C.}$  After the temperatures have become uniform, that of the water is found to be  $22^{\circ}\text{C.}$  Calculate the specific heat of the metal.

15. The temperature of 300 grammes of iron, whose specific heat is .113, is raised to  $98^{\circ}\text{C.}$ , and the iron is then immersed in 200 grammes of water originally at  $12^{\circ}\text{C.}$ , and contained in a copper vessel weighing 80 grammes. What will be the resulting temperature supposing no heat to escape, the specific heat of copper being .095?

16. A lump of platinum, weighing 12 lbs., was plunged into a vessel containing 10 lbs. of water at  $15^{\circ}\text{C.}$  When the temperature had become uniform it was found to be  $30^{\circ}\text{C.}$  Assuming that no heat was lost during the experiment, calculate what the temperature of the platinum must have been, its specific heat being .032.

17. A pound of iron at  $100^{\circ}\text{C.}$  is placed in a receiver of Laplace's calorimeter, and the water which flows from the apparatus is found to weigh .144 lbs. Find the specific heat of iron.

### CHAPTER III.

1. ENUMERATE the principal sources of heat.
2. Mention some cases in which chemical action produces a considerable quantity of heat.
3. Explain how the heat produced by an ordinary coal fire has been originally derived from the sun.
4. What condition must be fulfilled by the temperature of the products of combustion in order that a substance may con-

tinue to burn in air? Why will ammonia burn in pure oxygen but not in air?

5. How would you determine the calorific power of a sample of coal?

6. Mention some cases in which heat is produced by purely mechanical means.

7. What is the effect of heat upon stretched india-rubber? and what the effect on the temperature of a piece of india-rubber of suddenly stretching it?

8. Describe some experiment by which the heating effect of an electric current may be rendered apparent.

9. According to what law does the heat produced by an electric current in a wire depend upon the strength of the current and the resistance of the wire?

10. Account for the heat produced by the sudden crystallization of a quantity of salt which has been held in solution.

11. A pound of hydrogen in burning produces 34,000 units of heat. If it burn in pure oxygen at  $15^{\circ}\text{C.}$ , producing 9 lbs. of steam of specific heat  $\cdot 48$ , what should be the temperature of the flame if a suitable furnace prevent loss of heat by radiation?

12. In the preceding question, what would be the temperature of the flame if the hydrogen burned in a mixture consisting of 21 per cent. of oxygen and 79 per cent. of nitrogen, supposing the hydrogen capable of consuming the whole of the oxygen, and the specific heat of nitrogen to be  $\cdot 25$ ?

## CHAPTER IV.

1. DEFINE the coefficients of linear, superficial, and cubic expansion of a solid, and show that the coefficients of superficial and cubic expansion may be taken as twice and three times that of linear expansion respectively.

2. Place the following substances in the order of their coefficients of expansion: air, alcohol, glass, iron, mercury, water.

3. Describe Roy and Ramsden's method of determining the coefficient of linear expansion of a metal.

4. How is it proved that water has a point of maximum density, and what natural phenomenon does this account for?

5. What is the difference between the real and apparent coefficients of expansion of a fluid?

6. Show how to determine experimentally the coefficient of apparent cubic expansion of a liquid contained in a glass envelope.

7. Describe Dulong and Petit's mode of determining the absolute coefficient of expansion of mercury.

8. Explain how to employ a weight thermometer to determine the cubic expansion of a metal rod. Can the method be adopted with all metals?

9. Describe three forms of compensating pendulums.

10. Describe the plan adopted on railways in order to prevent variations of temperature changing the positions of points which are moved by levers at a considerable distance and connected to them by iron rods.

11. If a line of railway be laid with rails which are 8 yards long at  $0^{\circ}\text{C}$ ., find the least distance between consecutive rails which will allow of the expansion due to an increase of temperature of  $30^{\circ}\text{C}$ ., the coefficient of expansion of iron being  $\cdot 000012$ .

12. The cubic dilatation of mercury for  $1^{\circ}\text{F}$ . is about  $\cdot 0001$ , and the linear expansion of brass for  $1^{\circ}\text{F}$ . about  $\cdot 00001$ . A barometer with a brass scale correctly graduated at  $62^{\circ}\text{F}$ . reads 30 inches at  $47^{\circ}\text{F}$ . Find the true pressure reduced to inches of mercury at  $32^{\circ}\text{F}$ .

13. The coefficient of expansion of iron wire is  $\cdot 00001235$ , and the length of wire between a distant signal and the signal-box is 900 yards. If the wire has to be pulled through 6 inches to lower the signal, find what increase of temperature of the wire will allow the signal to return to danger after it has been lowered.

14. The capacity of the bulb of a thermometer is 500 times that of each inch of the tube; the mercury just fills the bulb at  $0^{\circ}\text{C}$ . How far up the tube will it stand at  $10^{\circ}\text{C}$ ., assuming the coefficient of cubic expansion of mercury in glass to be  $\cdot 000155$ ?

15. The specific gravity of silver at  $0^{\circ}\text{C}$ . is 10.5. Assuming its coefficient of linear expansion to be .00002, show that its specific gravity at  $1000^{\circ}\text{C}$ . is 9.90 (water at  $0^{\circ}\text{C}$ . being taken as the standard).

16. The time of vibration of a pendulum varies as the square root of its length. A clock keeps correct time at  $0^{\circ}\text{C}$ ., but loses 37.35 seconds per week at  $10^{\circ}\text{C}$ . Find the coefficient of linear expansion of its pendulum.

## CHAPTER V.

1. STATE Boyle's law, and describe some experiment by which its truth may be shown.

2. Define the coefficient of cubic expansion. What is its value in the case of the more permanent gases?

3. State the law connecting the pressure and temperature of a quantity of gas at constant volume.

4. Describe Balfour Stewart's apparatus for determining this relation.

5. What formula expresses the relation between the pressure and temperature of air at constant volume, and what the relation between the volume and temperature at constant pressure? Are the constant coefficients identical in the two formulæ?

6. State the relations between volume, pressure, and temperature in a perfect gas. In what manner does a real gas, such as carbonic anhydride, differ from the hypothetical perfect gas?

7. Describe some form of air thermometer, mentioning its advantages and disadvantages.

8. Do the indications of a mercurial thermometer, properly pointed at the freezing and boiling points, and uniformly divided, agree with those of an air thermometer for all temperatures for which it may be used? Why is the scale of the air thermometer considered of great importance?

9. 250 cub. cent. of hydrogen are measured at  $77^{\circ}\text{F}$ . and 750 mm. pressure; what would the gas measure at  $0^{\circ}\text{C}$ . and 760 mm. pressure?



10. If a gas occupy 1000 cub. cent. at  $10^{\circ}\text{C}.$ , what will be its volume at  $100^{\circ}\text{C}.$  under the same pressure?

11. A quantity of air which occupies  $1\frac{1}{2}\frac{2}{3}$  cub. ft. at  $10^{\circ}\text{C}.$  and under pressure of 30 inches of mercury, is raised to  $15^{\circ}\text{C}.$  and the pressure reduced to 29 inches. Find its volume.

12. Find the volume at  $45^{\circ}\text{C}.$  and under pressure of 1500 mm. of mercury, of a quantity of air which, at  $27^{\circ}\text{C}.$  and under a pressure of 760 mm., occupies 10 cubic ft.

13. A quantity of gas occupies 26 cub. in. at  $60^{\circ}\text{F}.$  and under a pressure of 29 inches of mercury; how much space will it occupy when the temperature has risen to  $65^{\circ}\text{F}.$  and the pressure to 50 inches?

14. If the compressed air in a flooded coal-pit occupied 2500 cub. ft. at  $50^{\circ}\text{F}.$  and under pressure of 70 inches of mercury, how much space would it fill at  $60^{\circ}\text{F}.$  and under pressure of 29.5 inches of mercury?

15. What will be the volume of 100 grammes of hydrogen at  $273^{\circ}\text{C}.$  under a pressure of 1420 mm., assuming that a litre of hydrogen at  $0^{\circ}\text{C}.$  under a pressure of 760 mm. weighs .0896 gramme?

16. If the air in a fire-balloon be raised to  $100^{\circ}\text{C}.$ , the temperature of the surrounding air being  $0^{\circ}\text{C}.$ , and the volume of the balloon 373 cubicfeet, show that the balloon will not ascend if its weight exceed  $7\frac{1}{2}$  lbs., assuming the weight of a cubic foot of air at  $0^{\circ}\text{C}.$  and under atmospheric pressure to be 1.2 ozs.

17. The volume of a bubble of gas generated under water at a depth of 200 fathoms is  $\frac{1}{3000}$  of a cubic inch. What will its volume be when it reaches the surface, the temperature remaining constant and the height of the water barometer being 33 feet?

18. A given quantity of gas kept at constant pressure, in being raised from  $0^{\circ}\text{C}.$  to  $100^{\circ}\text{C}.$  expands in volume in the ratio of 1 to 1.366. Find the absolute zero of the air thermometer on Centigrade scale.

19. Assuming that 30 cubic inches of air at  $0^{\circ}\text{C}.$  would occupy 41 cubic inches at  $100^{\circ}\text{C}.$  under the same pressure, find the position on the Centigrade scale of the absolute zero of the air thermometer.

## CHAPTER VI.

1. EXPLAIN the term "unit of heat," and "latent heat of fusion."
2. What becomes of the heat absorbed by a body in changing its state from the solid to the liquid, or from the liquid to that of vapour?
3. How is the latent heat of water determined?
4. Find the least quantity of water at  $0^{\circ}\text{C}$ . which, surrounding a pound of solid mercury at its freezing point,  $-39^{\circ}\text{C}$ ., will just melt the mercury without altering the temperatures of either substance, assuming the latent heat of fusion of water and mercury to be 79.5 and 2.8 respectively.
5. Find also the ultimate common temperature of the ice and mercury, their specific heats being  $\frac{1}{2}$  and  $\frac{1}{30}$  respectively.
6. What is the effect of pressure upon the melting-points of ice, paraffin, cast-iron, lead, gold, and type metal?
7. How do you explain the fact that two pieces of ice floating in contact on the surface of water will freeze together?
8. When is a liquid said to boil?
9. State the laws of evaporation, and explain the meaning of the assertion that one gas or vapour acts as a vacuum to another.
10. Explain the nature of ebullition. What circumstances influence the temperature at which it takes place?
11. How may the height of a mountain be measured by means of boiling water and a thermometer?
12. Define the quantity called *the latent heat of steam* at  $100^{\circ}\text{C}$ ., and describe a method by which it may be determined.
13. Water which is boiling freely in an open vessel is suddenly shut in by an air-tight lid; what follows?
14. In a closed vessel is contained water which has cooled so that ebullition has ceased. How may the water be made to boil again without applying heat to the vessel?

15. Regnault found the "total heat" of the vapour of water at  $t^{\circ}$  C. to be  $606.5 + 0.305t$ . Explain the meaning of this statement.

Water boils on the top of Mont Blanc at  $85^{\circ}$  C. Find the latent heat of steam there?

16. Describe a simple experiment showing that the pressure of aqueous vapour at a constant temperature cannot exceed a certain amount.

17. How would you expect a liquid to behave when heated to the boiling point, if the latent heat of evaporation were zero?

18. What is the dew-point, and how is it determined?

19. State Dalton's laws for the formation of vapour, and show how to verify them experimentally.

20. Describe Dalton's method of determining the tension of aqueous vapour between  $0^{\circ}$  C. and  $100^{\circ}$  C.

21. Explain the cooling action of a draught of air.

22. What is meant by the hygrometric state of the air? Explain how it is determined by means of Daniell's dew-point hygrometer.

23. What are the laws followed by saturated and non-saturated vapours respectively?

24. Describe the wet-bulb thermometer. What processes are going on in the air round the wet-bulb? What practical use is made of this thermometer?

25. Explain the action of Wollaston's cryophorus.

26. By what method has the increase of pressure of a given volume of air when raised from  $0^{\circ}$  C. to  $100^{\circ}$  C. been determined. How would the presence of water in the vessel affect the result of the experiment?

27. Explain how advantage may be taken of the spheroidal state of a volatile liquid in order to freeze water or mercury in a red-hot vessel.

28. How do you account for the apparently serrated form sometimes assumed by liquids when in the spheroidal condition?

29. Explain a process by which intense cold can be produced by means of carbonic anhydride?

30. Describe Dr. Andrews' experiments on the connection between the gaseous and liquid states in the case of carbonic anhydride.

31. Trace the changes in volume of a pound of ice as it is gradually heated under normal atmospheric pressure from  $-20^{\circ}\text{C.}$  to  $120^{\circ}\text{C.}$

What are the quantities of heat required during each successive rise of  $10^{\circ}\text{C.}$  during the process?

32. Seven pounds of iron at  $100^{\circ}\text{C.}$  are placed upon a mass of ice at  $0^{\circ}\text{C.}$  The specific heat of iron being  $\cdot113$ , and the latent heat of water  $79\cdot5$ ; how much water will be produced?

33. The latent heat of fusion of ice being  $79\cdot5$ , if 1 lb. of ice at  $0^{\circ}\text{C.}$  be dropped into 2 lbs. of water at  $15\cdot5^{\circ}\text{C.}$ , how much of it will remain unmelted?

34. If 10 lbs. of ice, originally at  $-10^{\circ}\text{C.}$  are boiled away at ordinary pressure, find the amount of heat required, having given that the specific heat of ice is  $\cdot5$ , the latent heat of fusion  $79\cdot25$ , and the latent heat of vaporization of water at  $100^{\circ}\text{C.}$  is 537.

35. Given that the latent heat of evaporation of water at  $100^{\circ}\text{C.}$  is 537, find how many pounds of steam at  $100^{\circ}\text{C.}$  must be blown into 50 lbs. of water at  $20^{\circ}\text{C.}$  to make the water boil.

36. If  $1\frac{3}{5}$  lbs. of ice at  $0^{\circ}\text{C.}$  be dropped into 1 lb. of water at  $100^{\circ}\text{C.}$ , what will be the resulting temperature of the mixture, and of what will it consist?

37. The specific heat of ice is  $\cdot504$ . The air and everything else in a certain room is at  $0^{\circ}\text{C.}$ , and a pound of ice at  $-10^{\circ}\text{C.}$  is placed in a vessel in the room, and water at  $12^{\circ}\text{C.}$  is slowly poured upon it. It is found that 7.045 lbs. of water are required to just melt the ice. Find the latent heat of fusion of water.

38. How much steam at  $100^{\circ}\text{C.}$  must be passed into 100 lbs. of water at  $0^{\circ}\text{C.}$  to raise the whole to  $50^{\circ}\text{C.}$ , supposing the whole of the steam condensed in the water?

39. What amount of steam at  $100^{\circ}\text{C}$ . must be passed into 100 lbs. of water at  $0^{\circ}\text{C}$ ., in which 10 lbs. of ice are floating, in order to raise the temperature of the whole to  $50^{\circ}\text{C}$ .?

40. How many pounds of iron at  $200^{\circ}\text{C}$ . must be placed in a vessel containing 10 lbs. of ice at  $-10^{\circ}\text{C}$ . in order to convert all the ice into steam at  $100^{\circ}\text{C}$ ., the capacity for heat of the vessel being equal to that of 2 lbs. of water, and no heat being supposed to escape?

41. The barometer stands at 30 inches, the thermometer at  $20^{\circ}\text{C}$ ., and the dew-point is  $15^{\circ}\text{C}$ . The maximum pressure of aqueous vapour at  $15^{\circ}\text{C}$ . is  $\cdot 508$  inch. What portion of the pressure indicated by the barometer is due to aqueous vapour?

42. What is the weight of 100 cubic centimetres of damp oxygen collected over water at  $20^{\circ}\text{C}$ . under a pressure of 760 mm., it being given that a litre of hydrogen at  $0^{\circ}\text{C}$ . and under a pressure of 760 mm. weighs  $\cdot 0896$  gramme: the densities of oxygen and aqueous vapour, compared with hydrogen as unity, are respectively 16 and 9, and the maximum pressure of aqueous vapour at  $20^{\circ}\text{C}$ . is 17.7 mm.?

43. Calculate the weight of a litre of hydrogen collected over water at  $20^{\circ}\text{C}$ ., the barometric pressure being 765 mm., assuming that a litre of hydrogen at  $0^{\circ}\text{C}$ . and under a pressure of 760 mm. weighs  $\cdot 0896$  gramme: the density of aqueous vapour is nine times that of hydrogen at the same temperature and pressure, and the maximum pressure of aqueous vapour at  $20^{\circ}\text{C}$ . is 17.4 mm.

44. A closed tube, 20 inches long, containing dry air at a pressure of 30 inches of mercury, is separated into two equal parts by a narrow piston. As much water as will evaporate is then introduced into one portion of the tube, the temperature of the whole being maintained at  $64^{\circ}\text{F}$ . How far will the piston move, the maximum pressure of aqueous vapour at  $64^{\circ}\text{F}$ . being assumed to be  $\cdot 6$  in.?

## CHAPTER VII.

1. FOR what purpose is iron comparatively useless in fire-proof structures, and why?

2. What is meant by *tempering*, as applied to steel?

How would you temper a graver for cutting iron?

3. What is the effect of heating and suddenly cooling a piece of copper?

What influence has hammering upon the mechanical properties of copper and brass?

4. What is the effect of heating a magnet (1) through three or four degrees, (2) to a red heat, and then allowing it to cool?

5. What influence has increase of temperature on the electrical resistance of (1) metallic conductors, and (2) electrolytes?

6. What is the Peltier effect?

Why is it generally not advisable to employ a very strong current in order to show the Peltier effect?

7. According to what law does the electro-motive force of a thermo-electric couple depend upon the temperatures of the junctions?

8. What is meant by *thermo-electric inversion*?

Describe an experiment illustrating this inversion.

9. Describe some form of thermo-electric pile.

## CHAPTER VIII.

1. GIVE a short description of the three methods by which heat is conveyed from one place to another. Distinguish carefully between convection and radiation.

2. How would you compare the thermal conductivities of two good conductors of heat?

3. Two equal bars of different metals are coated with wax, and one end of each exposed to the same source of heat. Why cannot we compare their thermal conductivities by observing the time required to melt the wax to a given distance along each?

4. Define the thermal conductivity of a substance, and show how it depends on the various fundamental units. Hence, find

the requisite factor for changing the measure of the thermal conductivity of a substance from pound, foot, minute, Fahr., into gramme, centimetre, second, C., supposing 1000 grammes to be equal to  $2\frac{1}{4}$  lbs., and 30 centimetres to one foot.

5. Give an account of Principal Forbes' investigation of the thermal conductivity of wrought iron at different temperatures.

6. How may it be shown that the conductivity for heat varies in different directions in the same crystal?

7. How may it be proved experimentally that water is a bad conductor of heat?

Explain how the temperature of water is raised when heat is applied beneath it.

8. Explain how buildings may be warmed by convection of heat in water.

9. Show what part is played by convection-currents in causing the north-west of Europe to be warmer than the places in the same latitude on the east coast of America.

10. At what temperature does water under ordinary circumstances attain its maximum density?

How may this temperature be experimentally determined?

11. The specific thermal conductivity of wrought iron, a second and an inch being the units of time and space, is .001. Calculate the number of units of heat transmitted per hour through each square foot of the plates of a steam-boiler constructed of  $\frac{3}{8}$  in. boiler plate, the temperature of the inner surface being kept at  $120^{\circ}$  C., and that of the outer at  $100^{\circ}$  C.

12. A certain boiler exposes 60 square feet of surface to the action of the fire. If the average thickness of the plates be  $\frac{3}{8}$  in., and the outside be maintained at  $140^{\circ}$  C., while the temperature of the inside is  $130^{\circ}$  C., find how many pounds of water can be evaporated per minute, the latent heat of steam at  $130^{\circ}$  C. being 516, and it being given that 6 units of heat pass per minute through an iron plate a foot square and an inch thick, when the temperature of one side is  $135.5^{\circ}$  C., and that of the other  $134.5^{\circ}$  C.

13. If  $S$  units of heat flow per second through an iron cube

of one foot edge when one face is maintained at a temperature always  $1^{\circ}$  C. above that of the opposite face, and no heat enters or leaves the remaining faces, how much heat will flow in one hour through a boiler-plate 4 feet long, 2 feet wide, and half an inch thick, when one face is in contact with boiling water and the other with melting ice, supposing the faces of the plates to possess the temperature of the water and ice respectively?

## CHAPTER IX.

1. DESCRIBE experiments which show that radiant heat obeys the same laws of reflection and refraction as light.

2. State your reasons for believing that the radiant energy which produces heat in bodies is of the same nature as light.

3. State Newton's law of cooling. Did the experiments of Dulong and Petit justify this law?

4. Describe an experiment which shows that good radiators are good absorbers.

5. By what experiments has the identity of the radiating and absorbing powers of bodies been established?

6. A blackened cubic vessel of boiling water is so placed that one surface radiates towards a plate of glass and another towards a plate of rock-salt. Describe the effects produced.

How would they be modified if the vessel were replaced by a body at a white heat?

7. How do you explain the apparent radiation of cold?

8. Explain fully what happens when a piece of ice is placed in one focus of a concave reflector and the bulb of a thermometer in the conjugate focus, the temperature of the room and of the thermometer being at first the same and higher than that of the ice.

9. Briefly describe the constituents of the radiation from the carbon points of an electric lamp when at a white heat.

10. What portions of the radiation can pass through rock-salt, crown-glass, and quartz, respectively?



11. Why does the air inside a green-house get so much heated by the sun?

12. In the Arctic regions the sun has been known to melt the pitch on a ship's side while the air has been much below the freezing-point. Explain this phenomenon.

13. How is it that ice can be formed at night in the Sahara?

14. Give a brief sketch of Prévost's theory of exchanges.

## CHAPTER X.

1. EXPLAIN the production of land and sea breezes.

2. How do you account for the Trade-winds?

3. Why is Moscow much colder in winter than Edinburgh?

4. Mention some means by which clouds may be produced.

How do you account for the heavy rain-fall in mountainous districts?

5. Does the presence of a cloud apparently clinging to the top of a mountain prove that there is no wind there?

How do you account for such clouds?

6. What is hoar-frost, and how is it produced?

Explain the difference between white and black frosts.

7. What is dew?

How is the deposition of dew affected by the state of the sky and the nature of the exposed surface?

## CHAPTERS XI. AND XII.

1. WHAT is meant by the energy of a system, and by the principle of the conservation of energy?

2. Distinguish between potential and kinetic energy.

3. State some of the arguments in favour of the dynamical nature of heat.

4. Describe the experiments which led Rumford to conclude that heat is motion.

5. Define the mechanical equivalent of heat.
6. Find how many units of heat are required to raise a mass of 80 tons to a height of 60 feet.
7. What assumption was made by Mayer in his determination of the Mechanical Equivalent of Heat?
8. Why does the specific heat of gas at constant pressure differ from its specific heat at constant volume?
9. How do Dr. Joule's experiments enable us to determine the specific heat of air at constant volume, its specific heat at constant pressure being known?
10. How has the relation between the specific heat of air at constant pressure and at constant volume been determined?
11. If a quantity of air be compressed without allowing heat to leave it, its temperature is raised; while if it be allowed to expand without receiving heat, its temperature falls. Explain this in accordance with the dynamical theory of gases.
12. Given the specific heat of gas at constant volume and its specific heat at constant pressure, show how to deduce the mechanical equivalent of heat.
13. Give a brief sketch of the molecular theory of gases, and in accordance with this theory explain why a gas becomes cooled when it expands under pressure.
14. What do you understand by a heat engine?
15. Explain the term *efficiency* as applied to heat engines, and state on what the efficiency of a reversible engine depends.
16. Give a very brief account of the principle of the dissipation of energy.
17. How much mechanical energy is necessary to melt a pound of ice at  $0^{\circ}\text{C}.$ ?
18. The specific heat of lead is  $\cdot 031$ , and its latent heat of fusion is  $5\cdot 37$ ; what is the mechanical equivalent of the heat required to raise 1 lb. of lead from  $300^{\circ}\text{C}.$  to its melting-point,  $326^{\circ}\text{C}.$ , and to melt it?
19. A leaden bullet strikes an iron target with the velocity which it would acquire in falling freely from rest through 12,500 feet. If the whole of the velocity of the bullet be

destroyed by the blow, and if 95 per cent. of the heat generated reside in the bullet, find the temperature to which it will be raised, its specific heat being  $\cdot 03$  and its initial temperature  $30^{\circ}$  C.

20. The melting-point of lead is  $326^{\circ}$  C., its specific heat is  $\cdot 03$ , and its latent heat of fusion  $5\cdot 37$ . What is the least height from which a piece of lead originally at  $20^{\circ}$  C. must be allowed to fall upon an inelastic plane in order to melt it, supposing 80 per cent. of the heat generated by the blow to be produced within the lead?

21. Given that 100 cubic inches of a certain gas at  $0^{\circ}$  C. and under a pressure of 15 lbs. weight per square inch weigh 30 grains, and that the specific heat at constant volume is  $\cdot 183$ , while the specific heat at constant pressure is  $1\cdot 42$  times this amount, find the mechanical equivalent of heat, assuming that the gas expands by  $\frac{1}{273}$  of its volume at  $0^{\circ}$  C. for each increment of  $1^{\circ}$  C. in temperature, the pressure being kept constant.

22. A certain steam-engine utilises 8 per cent. of the heat generated by the combustion of its fuel. The heat generated by the combustion of 1 lb. of coal will convert 16 lbs. of water at  $100^{\circ}$  C. into steam at the same temperature, the latent heat of steam being  $537$ . A horse can work for six hours a day at the rate of  $\frac{5}{6}$  of one horse-power. If coals cost 18s. per ton, and the maintenance of a horse 1s. per day, compare the expense of horse-power with that of steam, neglecting the wear of the engine and horse.

23. If the temperature of steam from the boiler of an engine of 10 horse-power be  $200^{\circ}$  C., and if the temperature of the condenser be  $80^{\circ}$  C., how many units of heat must leave the boiler per hour if the efficiency be one-fifth of that of a reversible engine working between the same limits of temperature?

24. What is meant by superheated steam, and what is the use of superheating steam in a high-pressure engine?

25. What is the use of a steam jacket round the cylinder of an engine?

26. How do you account for the freezing up of the ports of engines worked by compressed air?

The exhaust air from an engine driven by compressed air

sometimes produces a cloud of snow where it escapes from the exhaust port. Explain this.

27. What becomes of the latent heat of evaporation when water evaporates?

28. Describe the essential difference between a condensing and a non-condensing engine. What is a compound engine, and what are its special advantages?

29. How do you account for the efficiency of a good gas engine being greater than that of a steam engine?

30. What do you understand by the statement that the mechanical equivalent of heat is 424 kilogrammetres?

31. When one pound of hydrogen combines with eight pounds of oxygen it produces nine pounds of water; how many foot-pounds of work must be done in order to decompose one pound of water into its elements?

32. What is meant by the statement that the specific heat of saturated steam is negative? What practical bearing has this fact in connection with the steam engine?

33. Give a brief sketch of the dynamical theory of gases, explaining the cause of the difference between the specific heat of a gas at a constant pressure and its specific heat at constant volume.

34. Give a sketch of a normal indicator diagram of a non-condensing engine, working expansively and cutting off the steam at the end of one quarter of the stroke.

35. What is an isentropic line?

How are the isentropic lines of a gas related to the isothermal lines?

36. What is the difference between the expansion line on an indicator diagram when the cylinder is jacketed, and when no precautions are taken to keep up the temperature of the expanding steam?

37. What is the difference between the isothermal lines of  $20^{\circ}\text{C}$ . for a quantity of dry air and for the same quantity of air in the presence of water, it being given that the maximum pressure of aqueous vapour at  $20^{\circ}\text{C}$ . is 17.4 mm.?

What will be the form of the isothermal in the latter case?

38. Explain the principle of the indicator, and show how the indicator diagram may be utilised in determining the H. P. at which an engine is working. What information respecting the engine must be provided in addition to the indicator diagram?

39. What difference would you expect to find between the indicated H. P. of an engine, and that determined by applying a brake to the fly-wheel, and what is the cause of the difference?

40. How is the superheating of the steam in a locomotive engine generally effected?

41. The mean effective pressure of the steam on the piston of an engine possessing a single steam cylinder is 26 lbs. per square inch; the diameter of the piston is six inches, and the length of the stroke 12 inches; the engine makes 200 revolutions per minute. Find its indicated horse-power.

42. A quantity of air and a quantity of carbonic anhydride possess the same volume at  $15^{\circ}\text{C}$ . under ordinary atmospheric pressure. The pressure is then steadily increased to 40 atmospheres; make a sketch showing the difference between the isothermal lines for the two substances.

## MISCELLANEOUS EXAMPLES.

1. DISTINGUISH between saturated and unsaturated vapours. What is the effect of compressing unsaturated (or superheated) steam, (i) when its temperature is kept constant, (ii) when no heat is allowed to enter or leave it?

2. What is meant by the Thomson effect in thermo-electricity?

3. Why is it that a sprinkling of snow is sometimes observed to lie on the exposed surfaces of the sleepers of a railway, but not on the ballast?

4. What is the effect of heating (i) a piece of copper, (ii) a piece of steel to redness, and then plunging it in cold water?

5. How does the coefficient of expansion of water vary between  $0^{\circ}\text{C}$ . and  $100^{\circ}\text{C}$ .?

6. Explain why a deposit of moisture is sometimes formed on the walls of rooms heated by hot-water pipes, but not when the rooms are heated by open fires.

7. A quantity of water is distilled at a uniform rate of 5 lbs. per hour by water, which flowing uniformly enters the vessel surrounding the worm of the still at a temperature of  $10^{\circ}\text{C}$ ., and leaves it at a temperature of  $25^{\circ}\text{C}$ . Find the rate of flow of the water.

8. Trace the changes in volume which a block of ice at  $-10^{\circ}\text{C}$ . undergoes in being converted into steam at  $100^{\circ}\text{C}$ .

9. How do you account for the fact that when two equal bars of different metals are covered with wax, and heated equally at one end, sometimes the wax begins to melt first on one of the bars, but after a short time the melting proceeds most rapidly on the other bar?

10. How is the temperature of boiling water, and of the steam produced, affected by (i) a change of atmospheric pressure, (ii) a salt dissolved in the water, (iii) a stream of air-bubbles rising through the water, (iv) long boiling in a glass vessel?

11. State the conditions of evaporation. A closed vessel containing water and air at a pressure less than that of the atmosphere is raised to, and maintained at, the temperature of  $100^{\circ}\text{C}$ . What determines the cessation of boiling, and what the cessation of evaporation?

12. If the coefficient of cubic expansion of the liquid in a thermometer is less than that of the solid envelope, what effect will be produced on heating the instrument?

13. The moistness, or humidity, of the air does not depend solely on the amount of aqueous vapour present in it. Explain this statement, and point out the conditions on which the humidity of the air depends.

14. Calculate the weight of 1 litre of hydrogen collected over water at  $20^{\circ}\text{C}$ ., the atmospheric pressure being 765 mm., it being given that 1 litre of hydrogen at  $0^{\circ}\text{C}$ . and 760 mm. pressure, contains .0896 gramme, and the pressure of aqueous vapour at  $20^{\circ}\text{C}$ . is 17.4 mm.

15. Radiation from (i) an electric lamp, and (ii) a red-hot metal ball, is passed respectively through the following substances: a plate of glass, a solution of iodine in carbon disul-

phide, a plate of rock salt, a tube of dry air, and a tube containing a mixture of air and aqueous vapour. Describe the action of each of these media on the radiation from each source.

16. Water, if kept perfectly still, may be cooled several degrees below  $0^{\circ}\text{C.}$  without freezing, but if it be then agitated a portion assumes the solid state. Explain why the whole does not become solid.

17. A heated metal bar is placed upon a metallic support in the middle of a room. State all the different ways in which the bar loses heat, and carefully distinguish between them.

18. Describe how you would proceed in order accurately to determine the boiling point of a liquid.

19. How many pounds of carbon must be burnt in order to produce heat sufficient to convert 10 lbs. of ice at  $-5^{\circ}\text{C.}$  into steam at  $100^{\circ}\text{C.}$ , it being given that the heat of combustion of carbon is 8080 units?

20. Distinguish between the transmission of *temperature* and the transmission of *heat* along a bar.

21. Account for the laws of evaporation in accordance with the dynamical theory of the constitution of bodies.

22. Mention some bodies which assume a plastic condition in passing from the solid to the liquid state. Of what special value is this property in the case of those metals which possess it?

23. Supposing the section of the mercury tube in Bunsen's calorimeter to be one square millimetre and that the mercury undergoes a displacement of 5 centimetres after introducing into the calorimeter 4 grammes of iron at  $9^{\circ}\text{C.}$ ; find the specific heat of the iron.

24. Can hot water be raised to the same height as cold water by means of an ordinary pump? If not, what condition determines the height to which the hot water can be raised?

25. The specific gravity of air at  $0^{\circ}\text{C.}$  is  $\cdot 00129$ . A chimney 150 feet in height contains air at  $360^{\circ}\text{C.}$ , the temperature of the external air being  $15^{\circ}\text{C.}$  Find the pull of the chimney in inches of water-gauge supposing the air within it to be at rest.

## ANSWERS.

## CHAPTER I.

10.  $59^{\circ}$ .      11.  $36\frac{2}{3}^{\circ}\text{C}$ .      12.  $160^{\circ}\text{C} = 320^{\circ}\text{F}$ .  
 13.  $4\cdot5$ .

## CHAPTER II.

9.  $1274\cdot448\text{ lbs}$ .      10.  $16\frac{4}{11}^{\circ}\text{C}$ .      11.  $195\cdot5^{\circ}\text{F}$ , nearly.  
 12.  $\cdot1$       13.  $\cdot057$  nearly.      14.  $\cdot115$ .      15.  $24\cdot07^{\circ}\text{C}$ .  
 16.  $420\cdot6^{\circ}\text{C}$ .      17.  $\cdot114$  nearly.

## CHAPTER III.

11.  $7875^{\circ}\text{C}$ .      12.  $2885^{\circ}\text{C}$ .

## CHAPTER IV.

11.  $\cdot10368\text{ inch}$ .      12.  $29\cdot950$ .      13.  $14\cdot995^{\circ}$  nearly.  
 14.  $\cdot775\text{ inch}$ .      16.  $\cdot00001235\ldots$

## CHAPTER V.

9.  $226\cdot008\ldots\text{c.c}$ .      10.  $1318\cdot02\ldots\text{c.c}$ .      11.  $1\cdot0913\ldots\text{ft}$ .  
 12.  $5\cdot3707\ldots\text{c. ft}$ .      13.  $15\cdot23\ldots\text{c.ins}$ .      14.  $6048\cdot65\ldots\text{c. ft}$ .  
 15.  $1195\cdot66\ldots\text{litres}$ .      17.  $\cdot01245\text{ c. ins}$ .      18.  $-273\cdot2^{\circ}\text{C}$ .  
 19.  $-272\frac{8}{11}^{\circ}\text{C}$ .

## CHAPTER VI.

4.  $246\cdot54\ldots\text{grains}$ .      5.  $-25\cdot44^{\circ}\ldots\text{C}$ .      15.  $546\cdot425$ .  
 32.  $\cdot995\text{ lb}$ . nearly.      33.  $\frac{9\cdot7}{10\cdot5}\text{ lb}$ .      34.  $7212\cdot5$ .  
 35.  $7\cdot448\ldots$       36.  $0^{\circ}\text{C}$ . ;  $2\cdot2578\ldots\text{lbs}$ . water, and  $\cdot3421\ldots$   
 $\text{lbs. ice}$ .      37.  $79\cdot5$ .      38.  $8\cdot517\ldots\text{lbs}$ .      39.  $10\cdot724\ldots\text{lbs}$ .  
 40.  $657\cdot7$ .      41.  $\cdot01693$  of the whole pressure.      42.  
 $\cdot1322\ldots\text{gramme}$ .      43.  $\cdot099324\text{ gramme}$ .      44.  $\cdot099\ldots\text{inch}$ .

## CHAPTER VIII.

4.  $\frac{2}{31}$ .      11.  $27648$ .      12.  $11\frac{7}{3}\text{ lbs}$ .      13.  $69120000\text{ S}$ .

## CHAPTERS XI. &amp; XII.

6.  $7735\cdot2\ldots$       17.  $110157\cdot5\text{ foot-pounds}$ .      18.  $8584\cdot64$   
 $\text{foot-pounds}$ .      19.  $314\cdot7^{\circ}\ldots\text{C}$ .      20.  $25280\cdot625\text{ feet}$ .  
 21.  $1390\cdot02\text{ foot-pounds}$ .      22.  $12\cdot009\ldots : 1$ .      23.  
 $280757\text{ nearly}$ .      31.  $5251111$ .



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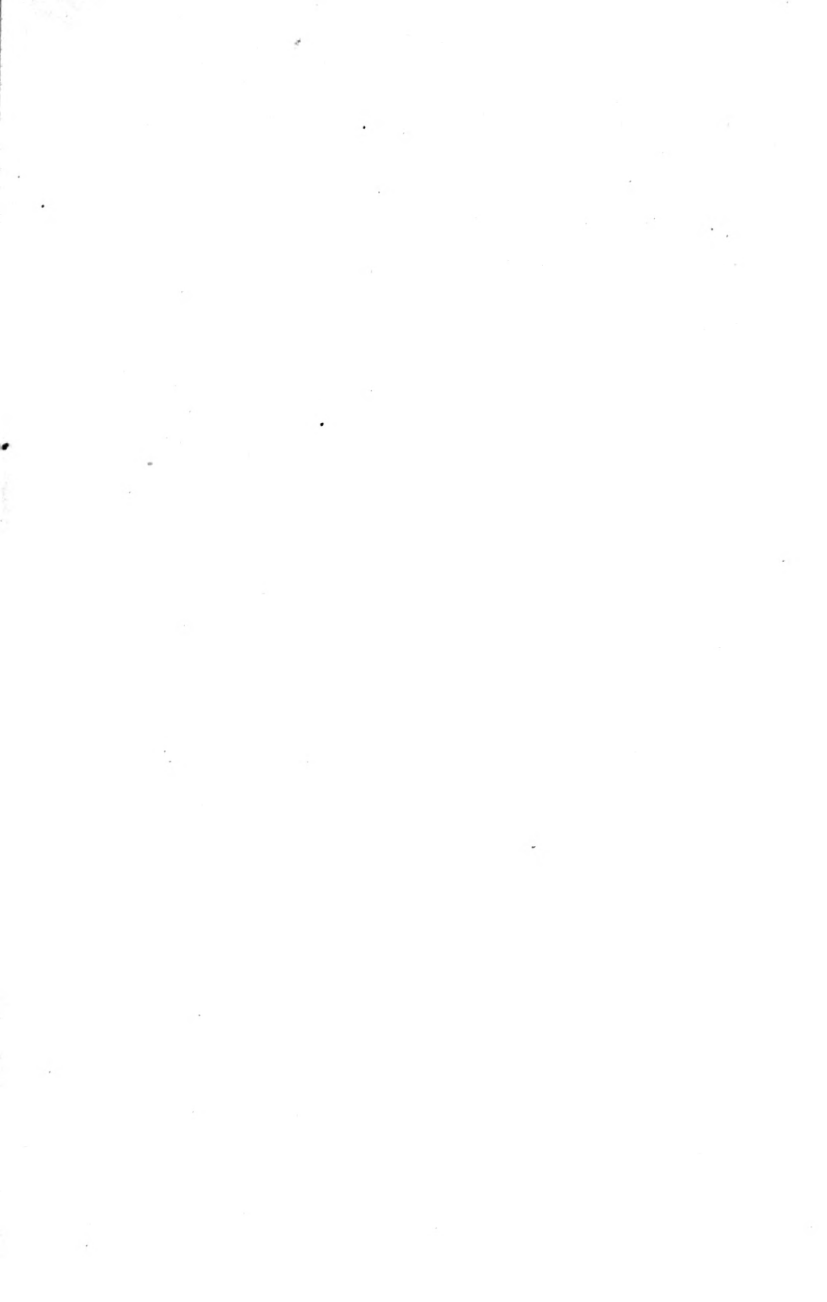
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